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\usepackage{amsfonts}

\usepackage{fancyhdr}

\usepackage{titlesec}

\usepackage{tocloft}

\usepackage{listings}

\usepackage[T1]{fontenc}

\usepackage{ascii}

\usepackage{graphicx}

\usepackage{pifont}

\usepackage{float}

\usepackage{sidecap}

\usepackage{wrapfig}

\usepackage{titletoc}

\usepackage{tocloft}

\usepackage[nottoc]{tocbibind}

\usepackage{afterpage}

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\usepackage{comment}

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\usepackage{balance}

\usepackage[table]{xcolor}

\usepackage{color}

\usepackage{amsmath}

%\usepackage[intlimits]{amsmath}

\usepackage{leftidx}

\usepackage[inference,shorthand]{semantic}

\usepackage{bm}

\usepackage{booktabs}

\usepackage{array}

\usepackage{tikz}

\usepackage[]{caption}

\usetikzlibrary{mindmap,trees}

\usepackage[UKenglish]{datetime}

\usepackage{acronym}

\usepackage{comment}

\usepackage[]{algorithm2e}

\usepackage{algorithmic}

\usepackage{subcaption}

%\usepackage[colorlinks=true,linkcolor=black]{hyperref}

\newcommand{\head}[1]{\textnormal{\textbf{#1}}}

\newcommand{\normal}[1]{\multicolumn{1}{l}{#1}}

\newcommand{\hilight}[1]{\colorbox{yellow}{#1}}

\addtocontents{toc}{\protect\renewcommand{\protect\cftchapleader}{\bfseries\protect\cftdotfill{\protect\cftdotsep}}}

\newenvironment{cmd}{\fontfamily{ascii}\footnotesize\selectfont}{}

\setcounter{secnumdepth}{3}

\setcounter{tocdepth}{3}

%\titleformat{\chapter}[hang]{\huge}{\thechapter}{1em}{}

%\titlespacing{\chapter}{0pt}{0pt}{1cm}

\titleformat{\chapter}[hang]

{\normalfont\huge\bfseries}{\chaptertitlename\ \thechapter:}{1em}{}

\pagestyle{fancy}

%\fancyhead{}

%\fancyfoot{}

\fancyhead[LE,RO]{\slshape \rightmark}

\fancyhead[LO,RE]{\slshape \leftmark}

\fancyfoot[C]{\thepage}

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%\footrulewidth 0 pt

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\renewcommand{\headrulewidth}{1.2pt}

%\renewcommand{\chaptername}{}

%\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_%%%%

\begin{comment}

%%%%%%%%%%%%% Preparing Oerview Diagrams %%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%

\usetikzlibrary{shadows,arrows,positioning}

% Define the layers to draw the diagram

\pgfdeclarelayer{background}

\pgfdeclarelayer{foreground}

\pgfsetlayers{background,main,foreground}

% Define block styles

\tikzstyle{materia}=[draw, fill=yellow!15, text width=10.0em, text centered,

minimum height=7.5em,drop shadow]

\tikzstyle{practica} = [materia, text width=10em, minimum width=12em,%(minimum width=2em) change the width of the yellow block

minimum height=5em, rounded corners, drop shadow]%(minimum height=5em) change the height of the yellow block

\tikzstyle{texto} = [above, text width=15em, text centered]

\tikzstyle{linepart} = [draw, thick, color=black!60, -latex', dashed]

\tikzstyle{line} = [draw, thick, color=black!240, -latex']

\tikzstyle{ur}=[draw, text centered, minimum height=0.01em]

\usetikzlibrary{fadings}

\usetikzlibrary{decorations}

\usepgflibrary{decorations.pathmorphing}

\tikzfading[name=fade out, inner color=transparent!0,

outer color=transparent!100]

% Define distances for bordering

\newcommand{\blockdist}{1.3}

\newcommand{\edgedist}{1.5}

%\newcommand{\etape}[2]{node (p#1) [etape]

% {#2}}

\newcommand{\Step}[2]{node (p#1) [practica]

{\\{ \Large\textit{#2}}}}

% Draw background

\newcommand{\background}[5]{%

\begin{pgfonlayer}{background}

% Left-top corner of the background rectangle

\path (#1.west |- #2.north)+(-0.3,0.5) node (a1) {};

% Right-bottom corner of the background rectanle

\path (#3.east |- #4.south)+(+0.3,-0.5) node (a2) {};

% Draw the background

\path[fill=gray!30,rounded corners, draw=black!50, dashed]

(-4,-8.5) rectangle (1,3);

\path[fill= gray!30,rounded corners, draw=black!50, dashed]

(1.8,-11.6) rectangle (6.8,-3.5);

%%%%%%%%%%%%%

%\fill[yellow!10!black] (-1,1) rectangle (4,3);

%\fill[yellow] (-4,-5.2) rectangle (4,1);

%\fill[inner color=blue!50!,outer color=blue!10!black] (4.7,-5.2) rectangle (12.5,1);

%%%%%%%%%%%%%%%%%%%%%%%%%%

\path (a1.east |- a1.south)+(3.5,0.5) node (u1)[texto]

{ \LARGE\textit{ #5}};

\path (#3.east |- #2.north)+(0,0.25)--(#1.west |- #2.north) node[midway] (#5-n) {};

\path (#3.east |- #2.south)+(0,-0.35)--(#1.west |- #2.south) node[midway] (#5-s) {};

\path (#3.east |- #2.north)+(0.7,0)--(#3.east |- #4.south) node[midway] (#5-w) {};

\end{pgfonlayer}}

\newcommand{\transreceptor}[3]{%

\path [linepart] (#1.east) -- node [center]

{\large #2} (#3);}

\end{comment}

%\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_%%%%

\usetikzlibrary{shadows,arrows,positioning}

% Define the layers to draw the diagram

\pgfdeclarelayer{background}

\pgfdeclarelayer{foreground}

\pgfsetlayers{background,main,foreground}

% Define block styles

\tikzstyle{materia}=[draw, fill=yellow!40, text width=10.0em, text centered,

minimum height=7.5em,drop shadow]

\tikzstyle{practica} = [materia, text width=8em, minimum width=6em,

minimum height=4em, rounded corners, drop shadow]

\tikzstyle{texto} = [above, text width=10em, text centered]

\tikzstyle{linepart} = [draw, thick, color=black!60, -latex', dashed]

\tikzstyle{line} = [draw, thick, color=black!50, -latex']

\tikzstyle{ur}=[draw, thick, text centered, minimum height=0.01em]

\usetikzlibrary{fadings}

\usetikzlibrary{decorations}

\usepgflibrary{decorations.pathmorphing}

\tikzfading[name=fade out, inner color=transparent!0,

outer color=transparent!100]

\tikzset{>=latex}

% Define distances for bordering

\newcommand{\blockdist}{1.3}

\newcommand{\edgedist}{1.5}

%++++++++++++++++++++++++++++++++++++++++

%\newcommand{\practica}[2]{node (p#1) [practica]

%{Pr\'actica #1\\{\scriptsize\textit{#2}}}}

\newcommand{\Step}[2]{node (p#1) [practica]

{\\{ \Large\textit{#2}}}}

%++++++++++++++++++++++++++++++++++++++++

% Draw background

\newcommand{\background}[5]{%

\begin{pgfonlayer}{background}

% Left-top corner of the background rectangle

\path (#1.west |- #2.north)+(-0.3,0.9) node (a1) {};

% Right-bottom corner of the background rectanle

\path (#3.east |- #4.south)+(+0.5,-0.35) node (a2) {};

%++++++++++++++++++++Two backgrounds++++++++++++++++++++

% Draw the background

\path[fill=gray!20,rounded corners, draw=black!50, dashed]

(a1) rectangle (a2);

% \path[fill=blue!20,rounded corners, draw=black!50, dashed]

%(-4,-5.2) rectangle (4,1);

%++++++++++++++++++++++++++++++++++++++++

\path (a1.east |- a1.south)+(1.9,-0.57) node (u1)[texto]%place of the text above the diagram

{\Large\textit{ #5}};

\path (#3.east |- #2.north)+(0,0.25)--(#1.west |- #2.north) node[midway] (#5-n) {};

\path (#3.east |- #2.south)+(0,-0.35)--(#1.west |- #2.south) node[midway] (#5-s) {};

\path (#3.east |- #2.north)+(0.7,0)--(#3.east |- #4.south) node[midway] (#5-w) {};

\end{pgfonlayer}}

\newcommand{\transreceptor}[3]{%

\path [linepart] (#1.east) -- node [above]

{\scriptsize #2} (#3);}

%%%%%%%%%%%%%%%%%% BEGING Document %%%%%%%%%%%

\begin{document}

%%%%%%%%%

%%%%%%%%%%%%%%%%%%%% Title page %%%%%%%%%%%%%

%%%%%%%%%

\begin{titlepage}

\vspace\*{\fill} % Vertical Align

\begin{center}

% LOGO SECTION

%----------------------------------------------------------------------------------------

\includegraphics[width=0.4\textwidth]{logo.jpg}\\[2.4cm]

%----------------------------------------------------------------------------------------

% Title

\textbf{\LARGE Platform for Microrobot Navigation} \\ [1.5cm]

% Author and date

\textbf{\LARGE Nafiseh Vahabi} \\ [1.3cm]

% Report title

\textbf{\normalsize A thesis submitted in partial fulfilment of the requirements for the degree of } \\ [0.2cm]

\textbf{\normalsize MRes in Medical Robotics and Image Guided Intervention and for the Diploma of}\\ [0.2cm]

\textbf{\normalsize Imperial College}\\ [2.3cm]

\textbf{\LARGE Imperial College London} \\ [0.6cm]

\textbf{\large Department of Surgery and Cancer} \\ [1.90cm]

\newdateformat{UKvardate}{%

\monthname[\THEMONTH], \THEYEAR}

\UKvardate

\textbf{\large \today} \\ [1.9cm]

% Examiner details

\textbf{\large Dr. Henry Ip} \\ [0.3cm]

\textbf{\large Dr. Vincenzo Curto} \\ [0.3cm]

\textbf{\large Prof. Guang-Zhong Yang} \\ [1.5cm]

\end{center}

\vspace\*{\fill}

\end{titlepage}

%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%END Title page %%%%%%%%%%%%%

%%%%%%%%%

\pagenumbering{roman}

%\section{Acknowledgements}

%\addcontentsline{toc}{chapter}{\numberline{}Acknowledgements}

%\pagenumbering{roman}

\setcounter{page}{1}

\thispagestyle{plain}

\chapter\*{Acknowledgements}

First and foremost, I wish to express my sincere thanks to my supervisor Prof. Yang for his valuable guidance

and high level supervision during my MRes course.

\paragraph{}

I am especially grateful to my supervisors Dr. Henry Ip and Dr.Vincenzo Curto for their assistance

and suggestions throughout this project. My thanks to both of you for helping me to improve my final presentation,

reading the draft of my thesis and providing me with your valuable feedback.

\paragraph{}

I would like to thank Dr. Ebubekir Avci for helping me to take images of my fabricated structures with

Scanning Electron Microscope.

\paragraph{}

I would like to thank the course leader Dr. Daniel Elson for all his organisation and arrangements

for the MRes students to make this course an enjoyable one.

\paragraph{}

I am most grateful to my husband Ali for all his love, support and and help during

my study. Thanks for advising me advising me with my , for your patience

and most importantly caring for our children during this pressured time. This thesis would not

have been possible without you.

\paragraph{}

I would like to thank my daughter Parmin and my son Ario who always give me their love and joy.

\paragraph{}

Finally, my special thanks to my parents and my brother for all their encouragement and support during the project.

\pagebreak

\renewcommand{\cftsecleader}{\cftdotfill{\cftdotsep}}

\tableofcontents

%\setcounter{secnumdepth}{2}

%\setcounter{page}{1}

\pagebreak

%\addcontentsline{toc}{chapter}{\numberline{}List of Figures}

\setcounter{page}{3}

%\thispagestyle{plain}

\listoffigures

%\chapter\*{List of Figures}

\pagebreak

%\addcontentsline{toc}{chapter}{\numberline{}List of Tables}

\setcounter{page}{4}

%\thispagestyle{plain}

\listoftables

%\chapter\*{List of Tables}

\pagebreak

%%%%%%%%%%%%%%%%% List of ACronyms %%%%%%%%%%%%%%%

\setcounter{page}{5}

\chapter\*{List of Acronyms}

\begin{acronym}

\addcontentsline{toc}{chapter}{\numberline{}List of Acronyms}

\acro{SBT}{Slender Body Theory}

\acro{FBMS}{Fixed-beam Moving-sample}

\acro{gwl}{General Writting Language}

\acro{MPC}{Magnetic Polymer Composite}

\acro{MBFS}{Moving-beam Fixed-sample}

\acro{RFT}{Resistive Force Theory}

\acro{RSM}{Regularized Stokeslet Method}

\acro{SEM}{Scanning Electron Microscope}

\acro{stl}{STereoLithography}

\end{acronym}

\pagebreak

%------------------------------------------- Abstract -------------------------------------

\begin{abstract}

%\addcontentsline{toc}{chapter}{\numberline{}Abstract}

%abstract

Microrobots provide the opportunity to develop a system, the size of a bacteria, capable of swimming in a controllable manner

in a high viscous fluid and which could perform delicate tasks such as targeted drug delivery in a medical application. Furthermore,

microrobots have benefited from fabrications technology, which makes them biocompatible.

The structures and functions of microorganisms make them a suitable reference point for the design of

microrobots. This is because the flagella propulsion of microorganisms such as E.coli can be used to provide

an efficient approach for modelling micro swimmers locomotion method in low Reynolds number regime.

Microrobots demonstrated precise and controllable movements under low strength magnetic field.

In this study we reviewed a range of bio mimetic microrobots in terms of their design, fabrication, propulsion

method in the fluid environment.

The key characteristics of the helical shape microswimmers were optimised and the new design of helical

microrobot is presented. The new design is demonstrated the microrobot with the variable pitch that is

satisfied the fabrication requirement.

The three propulsion methods studied were; Resistive Force Theory, Regularised Stokeslet Method

and Slender Body Theory. The last two methods have not previously been used

for remotely controlled microrobots. We provide the simulation platform for the

swimming microrobot in a high viscose fluid. The simulation algorithm takes a desired translational velocity of a microrobot

and calculates the electric current required to generate a dynamic magnetic field.

%\pagenumbering{roman}

\setcounter{page}{6}

\thispagestyle{plain}

\end{abstract}

%-------------------------------------------END Abstract -------------------------------------

\pagenumbering{arabic}

%%%%%%%%%

%%%%%%%%%%%%%%%%%%%% Introduction %%%%%%%%%%%%%

%%%%%%%%%

\chapter{Introduction}

Robotic surgery has demonstrable advantages such as reducing pain and discomfort and

minimising scaring after invasive surgery. As robotic technology continues to move towards miniaturisation, the idea of using

a microrobot for medical applications such as cardiovascular surgery, also becomes more attractive~\citep{phil2013robotics}

The potential applications of a microrobot can be classified into two main categories; in vivo and in vitro applications.

Some in vivo examples are drug delivery, delivery of hyperthermia for cancer treatment and ablation of material.

In vitro applications can involve microfluid control, cell characterisation and manipulation~\citep{edd2003biomimetic}.

\paragraph{}

Magnetically actuated helical microswimmers have been reported as a safe microdevice for biomedical applications

~\citep{peyer2013magnetic}. However, there are number of challenges such as the issue of biocompatibility and the material

used to make a microrobot, which needs to be considered carefully for in vivo application\citep{qiu2014noncytotoxic}.

Research into swimming microrobots has identified two primary difficulties, namely the power source and a suitable locomotion method.

This is because

there are many cells, proteins and fibres in biofluid that prevent the motion of the microrobots~\citep{peyer2013bio}.

The extremely small size of the microrobot and the

complex biofluid environment makes the design aspect a very challenging one.

Furthermore, the design of a microrobot will depend on its application and the desired task to be performed.

Artificial bacteria flagella is a popular microrobot inspired by nature~\citep{qiu2014noncytotoxic}.

They have helical shape, are composed of magnetic material and are capable of mimicking the three-dimensional

motion of the bacteria in a high viscous fluid.

\paragraph{}

The reason for developing a simulation platform for microroswimmers navigation is to analyse

their characteristic and behaviour under low magnetic field. A few numbers of parameters

has a key role on designing the helical shape microswimmer. The simulation

challenge can be divided into two main parts; propulsion mechanism of the microrobot and actuation method. The

popular method, \ac\*{RFT}, describes the motion of the helix object in a low number regime and

is applied by most helical microswimmers\rq{}s algorithms\citep{qiu2014noncytotoxic}. In this

project, we studied two more propulsion methods for the remotely controlled helical microswimmers

in a dynamic magnetic field, \ac\*{RSM} and \ac\*{SBT}. An actuation method of a magnetic microrobot

can be either force driven or

torque driven~\citep{qiunanohelices}. Fabrication of a microdevice has been a considerable challenge

for some time ~\citep{qiunanohelices}, which has now been

overcome by fabrication methods such as 3D laser lithograph.

\paragraph{}

In summary, part of the algorithms developed for the propulsion mechanism of the microswimmers

and their actuation technique, were based on \citeauthor{mahoney2011velocity}\rq{}s research. Two new

propulsion methods for the helical microswimmers were used in the \citeauthor{rodenborn2013propulsion}\rq{}s

work. This involved a reversible speed-variable motor such as a micro metal gear motor being used to rotate

the microswimmer. In this study, these two propulsion mechanisms were applied to the microswimmers that

are then controlled remotely by a magnetic field. Therefore, the algorithm has taken the desired translational

velocity as an input and produced the rotational velocity as an output. By knowing the rotational velocity,

we will be able to compute the electric current required to produce the desired translational velocity.

In addition, the new design is developed and printed for the helix using a variable pitch

in the helix design rather than a constant pitch to satisfy

the fabrication process.

%%%%%%%%%%%%

%%%%%%%%%%%%%%%%% OVERVIEW DIAGRAM %%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%

\begin{figure}

\centering

% \begin{subfigure}{1.0\textwidth}

\includegraphics[width=\textwidth]{overview\_Dig1}

% \caption{System architecture}

%\label{System architecture}

% \end{subfigure}

\caption[Project workflow]{Project workflow. The project started by designing microhelix and followed by parallel work on the

fabrication and simulation. The final structures were analysed and optimised in terms of their design.}\label{overview\_Dig1}

%add desired spacing between images, e. g. ~, \quad, \qquad, \hfill etc.

%(or a blank line to force the subfigure onto a new line)

%add desired spacing between images, e. g. ~, \quad, \qquad, \hfill etc.

%(or a blank line to force the subfigure onto a new line)

\end{figure}

\begin{comment}

\begin{figure}

\centering

\begin{tikzpicture}[<->,scale=1.25,transform shape]

% Draw diagram elements

\path \Step{1}{Propulsion Algorithm};

\path (p1.south)+(0.0,-2.0) \Step{2}{Actuation Algorithm};

\path (p2.south)+(0.0,-3.5) \Step{3}{Nanoscribe};

\path (p3.south)+(0.0,-2.0) \Step{4}{SEM};

\path (p4.east)+(3.0,5.5) \Step{6}{Optimisation};

\path (p6.west)+(-8.0,0.00) \Step{5}{Design};

%\path (p4.east)+(5.0,0.0) \practica{7}{Calculate forward and inverse kinematic};

%\path (p5.east)+(5.0,0.0) \practica{8}{Design feedback control mechanics};

%\path (p5.south)+(3.5,-2.0) \practica{9}{Integrating e-AR and robot};

%\path (p9.south)+(0.0,-1.5) \practica{10}{Laparoscopy Robot};

%\draw [>=stealth,red] (0,.6) -- +(1,0);

%\draw [blue] (0,.3) -- +(1,0);

%\draw (0,0) -- +(1,0);

% Draw arrows between elements

\path [line] (p1.south) -- node [above] {} (p2);

\path [line] (p3.south) -- node [above] {} (p4);

\path [line] (p5.north) -- node [above] {} (p1);

\path [line] (p5.south) -- node [above] {} (p3);

\path [line] (p4.east) -- node [above] {} (p6);

\path [line] (p2.east) -- node [above] {} (p6);

%\path [line] (p5.south) -- node [above] {} (p9);

%\path [line] (p8.south) -- node [above] {} (p9);

% \path [line] (p9.south) -- node [above] {} (p10);

\background{p1}{p1}{p2}{p2}{Simulation }

\background{p3}{p3}{p4}{p4}{Fabrication}

% \path [line] (p5.south) -- node [above] {} (bk3-n);

% \path [line] (bk3-s) -- node [above] {} (p8);

% \path [line] (bk3-s) -- node [above] {} (p9);

%\path (bk1-e)+(+6.0,0) node (ur1)[ur] {};

% \path (bk2-w)+(+6.0,0) node (ur2)[ur] {};

%\path (bk3-w)+(+3.0,0) node (ur3)[ur] {};

% \transreceptor{bk1-e}{pre processing}{ur1};

% \transreceptor{bk2-w}{Feature selection}{ur2};

%\transreceptor{bk3-w}{classification}{ur3};

%%%%%%%%%%%%%% CHANGED%%%%%%%%%%%%%

\end{tikzpicture} %

\caption{System Architecture.} %

\label{System Architecture} %

\end{figure} %

%

%%%%%%%%%%%%%% CHANGED%%%%%%%%%%%%%

\end{comment}

\paragraph{}

A literature review on the different aspects of microrobots is presented in chapter 1.

An overview of the main microrobot designs are summarised in the table \ref{Micro}.

Section \ref{microDesign} demonstrates the details of the effective parameters on

microrobot\rq{}s design and optimises them to characterise the new design for

a helix shaped microrobot. The major part of this project involves studying, solving and

implementing the simulation methods

described in section \ref{simulation}.

Whilst the mechanism of both methods are explained in the section \ref{microActuation}, we

only implemented the torque driven actuation method in this study.

Section \ref{fabrication} presents a brief history of the fabrication techniques and section \ref{microFabric}

describes the fabrication method applied in this study. Chapter 3 provided the results of this work in terms of both

simulation and fabrication. The key issues are discussed in chapter 4 and conclusion and potential

future work is described in chapter 5. An overview of the entire system is shown in diagram \ref{overview\_Dig1}.

%%%%%%%%%

%%%%%%%%%%%%%%%%%%%% literature review %%%%%%%%%%%%%

%%%%%%%%%

\section{Literature review}

\subsection{Bioinspired microrobots}

One of the most challenging aspects of designing a robot on a very small scale such

as a nanorobot is simplicity. The reason is, integration between various components

will become unfeasible on such a small

scale if the design is complex. Hence the development of the nanorobot or even microrobot

should be based on the essential functionality, avoiding any unnecessary components~\citep{gao2013bioinspired}.

By learning from nature and mimicking the structure of live organisms, the successful

scientific applications were created~\citep{qiunanohelices}. The following section describes a

few examples of swimming microrobots that were imitated from nature.

\paragraph{Reynolds number}

%\begin{figure}

% \centering

\begin{wrapfigure}{r}{0.5\textwidth}

\begin{center}

\includegraphics[width=0.5\textwidth]{10}

\caption[Propulsion mechanism of mastigonemes flagellum]{ The propulsion mechanism of the smooth flagellum and a mastigonemes flagellum.

The propulsion direction of smooth flagellum (top design) is opposite of flagella\rq{}s propagation

wave (second from the top). Their artificial design (blue structures) is based on

their locomotion mechanism~\citep{gao2013bioinspired}.}

\label{10}

\end{center}

\end{wrapfigure}

%\end{figure}

To understand how micro-organisms swim in a fluidic environment, it is essential to study their propulsion

mechanism. In the fluidic regime the Reynold number (Re) has a substantial effect on a microdevice

locomotion~\citep{peyer2013magnetic}. The Reynolds number describes the ratio of the inertial forces versus viscous

forces according the following formula;

\begin{equation}

Re = \cfrac{UL\rho}{\mu}

\label{eq:4}

\end{equation}

Where $ U$ is velocity, $L$ is characteristic length, $\rho$ is the density and $\mu$ is viscosity of the fluid.

\subsubsection{Flagella style microrobots}

Helical flagella and cilia are two well-known microswimers in nature that have had their functionality employed

for motion generation in artificial microrobots (Figure~\ref{cilia}) ~\citep{gao2013bioinspired}.

\begin{figure}

\centering

\includegraphics[width=0.9\textwidth]{cilia}

\caption[Micro-structures and microdevices]{Micro-structures and microdevices. The illustration of both flagellum and cilia shapes and microdevices mimicked the flagellum and cilia

structures~\citep{peyer2013bio}.}

\label{cilia}

\end{figure}

\paragraph{}

In 2007, Bell~\citep{gao2013bioinspired} presented the first artificial bacteria flagellum microrobots and then

Zhang characterised them in 2009~\citep{gao2013bioinspired}. This microrobot was formed of two

components; a rigid helical tail and a soft magnetic metal head.

The head diameter

was $2.8 \mu m$ and its length was $30-100 \mu m$. Since then, other scientists proposed a slightly different design

, that mostly have the rigid helical tail shape. However, in some cases the magnetic

materials is used in the tail of the device rather than the head~\citep{gao2013bioinspired}.

The helical rotation of flagella and the travelling wave beat of cilia are two non-reciprocal propulsion

mechanisms in microorganisms. Mimicking a rotating flagellum at low Reynolds number to generate an

adequate torque to overpower the high viscous drag requires two main elements; a rotary motor and a

power source~\citep{qiunanohelices}.

%\begin{figure}

%\begin{SCfigure}

%\centering

\begin{wrapfigure}{r}{0.5\textwidth}

\begin{center}

\includegraphics[width=0.4\textwidth]{HelixShapes}

\caption[Helical microswimmers design]{Three design of helical microswimmers~\citep{peyer2013magnetic}.}

\label{HelixShapes}

\end{center}

\end{wrapfigure}

%\end{SCfigure}

%\end{figure}

An electromagnetic rotary motor can be used in designing a helical flagella style microrobot that

requires a considerable current. However piezoelectric rotary motors are an alternative option

that are appropriate for miniaturisation but necessitate high input voltage. Hence, designing a microrobot with a

combination of an onboard power source and a motor is a challenging task~\citep{qiunanohelices}.

Another design of microswimmers was inspired by the function of magtigonemes in nature~\citep{tottori2013artificial}.

A smooth flagellum propels against the direction of the flagella\rq{}s propagation wave. However,

the flagellum covered by magtigoneme propels in the same direction as the flagellum wave (Figure~\ref{10}). Mimicking

the structure of flagellum and using 3D lithography and electron beam evaporation formed the fabrication

method in these microswimmers.

The anisotropic viscous drag on the flagella is an important fact for locomotion in low Reynolds number fluid.

Flagella movement in the opposite direction of the flagella wave is because the

viscous drag coefficient perpendicular to the flagella is greater than the viscous drag coefficient parallel to

the flagella~\citep{tottori2013artificial}.

The artificial smooth flagellum is powered by an external magnetic field.

The rotating field, i.e. rotational frequency, field strength and angles that

defined the rotational axis is controlled by the current in the external coil. The helical microrobots rotate

synchronously with the rotation of the magnetic field and move forward and backward accordingly~\citep{tottori2013artificial}.

The displacement of the microswimmer along the rotational axis can be measured and the result

used to calculate the average velocity of the swimmers. There is a linear relationship between an input

field frequency and swimming speed. According to their result~\citep{tottori2013artificial}, a propulsive force generated by

the mastigoneme is in opposite direction of the force generated by the main helical filament.

However, this velocity is only valid if the external force is zero. The proposed

design~\citep{tottori2013artificial} is rigid and an external stimulus may be used to regulate the swimming

speed and direction if the swimmer can fold and unfold their structure.

There are three common shapes of microrobots

based on the rotary action; a helix, a screw and a twisted ribbon shape around its

axis (Figure~\ref{HelixShapes}). For the purpose of drilling into solid matter such as biological tissue the screw and helix

design would be more appropriate. The rotational motion of helical micro

swimmers is one of the most effective propulsion methods in the low Reynolds number scenarios

because it leads to translational motion. Microrobots with the microspheres structure perform similarly

to the helical swimmers and are capable of swimming in the flowing liquid within the micro-fluidic channel~\citep{kim2013fabrication}.

There are two main factors that affect the movements of the microrobot in the external magnetic

field; low coercivity and high saturation magnetization. Also, the motion of the microrobot is related to

its size given the same magnetic field strength and as such, by increasing the size of the microrobot with the inflexible magnetic material

volume, the velocity will decrease ~\citep{kim2013fabrication}.

The surface friction and the drag forces are two resistive forces that impede the microrobot\rq{}s

motion. Hence, the input magnetic force must be sufficient to overcome these forces for microrobot

manipulation. Furthermore, the weight of the microrobot requires gravity compensation in the z-direction by

the magnetic field. The navigation methodology should compensate for gravity to avoid sinking and enable velocity to be

controlled wirelessly. \citeauthor{mahoney2011velocity} described an algorithm for helical microswimmers velocity

control plus gravity compensation. In the proposed model the correct pitch angel and

rotation speed is calculated to achieve the commanded velocity (Figure~\ref{11}).

\begin{figure}

\centering

\includegraphics[width=0.9\textwidth]{11}

\caption[Effect of gravity on swimming microrobot]{The effect of the

gravity on the microrobot motion direction and gravity compensation~\citep{mahoney2011velocity}.}

\label{11}

\end{figure}

A magnetic field can be used for controlling teams of microrobots as well as a single

one. \citeauthor{kim2013fabrication} proposed a method that used a combination of two magnetic materials to

attain on/off magnetization of each microrobot. The overall control of the group of microrobots

was achieved by managing the magnetization state of each microrobot. In addition, a second technique has been

developed for three-dimensional motion of the team of microrobots in a fluidic environment. In

the latter method, each microrobot is designed in such a way that it uniquely responds to the

input magnetic field. Therefore, several microrobots can provide feedback position control in

3D system~\citep{kim2013fabrication}.

An untethered spherical magnetic micromanipulator creates a locally induced rotational fluid flow gradient.

The created rotational flow propels micro-objects in the flow area. A team of microrobots could perform

a complex task in micro-transport and micro-assembly~\citep{kim2013fabrication}.

In another study ~\citep{tottori2012magnetic}, a helical microrobot was designed to swim in a low Reynolds number.

Two designs are selected to run the experiment; the first one is a bare helical structure and the second one is the

helical shape with the microholder attached at the end. Both designs will generate the corkscrew

motion in a fluid environment when the magnetic filed is about few mili Tesla. The second

design (device with the microholder) is capable of transporting a microobject accurately to the

target ~\citep{tottori2012magnetic}.

In ~\citeauthor{tottori2012magnetic}\rq{}s study eight designs of microrobots were proposed and tested.

The uniform static magnetic field was used to explore the magnetic shape anisotropy and the

magnetic actuation was monitored in the rotating magnetic field. In the static magnetic field the

set of microrobots had helical angles $\theta$ ranging from ${45^{\circ}}$ to ${70^{\circ}}$ when suspended in the deionised water.

%\begin{figure}

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\begin{wrapfigure}{r}{0.5\textwidth}

\begin{center}

\includegraphics[width=0.5\textwidth]{7}

\caption[Effect of frequency on microswimmer\rq{}s behaviour]{(a) The misalignment of helical angle $\theta$ with an angle

between megnetic field and helix central axis. (b) the oscillation behaviour

of the microswimmer with the high and low frequencies~\citep{tottori2012magnetic}.}

\label{ref7}

\end{center}

\end{wrapfigure}

%\end{figure}

This showed (Figure~\ref{ref7}) that a smaller helix angle $\theta$ results in a reduced misalignment

angle $\alpha$ because microrobots longest axes will be aligned to the direction of the external magnetic field.

However in a helical microrobot with larger helix angles ($\theta$), the magnetization direction would change to

the radial axes of the helix ~\citep{tottori2012magnetic}.

In the rotating magnetic field, the micro helical swimmer exhibits different behaviours depending on

the strength of the applied frequency in the fixed magnetic field. At low frequencies the micro helix oscillated

around the helical axes, however the oscillating behaviour changed to the

corkscrew motion after increasing the applied frequency in the magnetic field. This is similar to characteristics of

microrobots with an incorporated

microholder~\citep{tottori2012magnetic}.

The velocity of helical micro swimmers depends on their size and shape. A linear relationship was

observed between the input frequencies and swimming velocity of the micro swimmers. The outcome of

the comparison between three microhelixs with the same helix angles showed that the microhelix with the

greatest diameter has the highest speed, in accordance with the following formula;

\begin{equation}

U = {\cfrac{(C\_n - C\_1) \sin \theta \cos \theta}{2(C\_n \sin^2 \theta + C\_1 \cos^2 \theta)}} \big( d \varpi \big)

\end{equation}

Where $C\_n$ is a drag coefficient perpendicular to the filament and $C\_1$ is a drag coefficient

parallel to the filament. $ \varpi$ is the rotational frequency and $d$ is the rotational diameter of

the helix ~\citep{tottori2012magnetic}.

\paragraph{}

The important role of helix angle in the magnetization structure of helical micro swimmers

was confirmed by \citeauthor{peyer2013bacteria} \citep{peyer2013bacteria}, who used direct laser writing (DLW) as a fabrication method

on a \ac\*{MPC}. The \ac\*{MPC} are non-cytotoxic and showed

super paramagnetic characteristic because magnetic material was already included in the polymer.

The relationship between the torque $T$, the drag force $F$, the object\rq{}s velocity $\nu$ and rotational

speed $\omega$ is linear and modelled by $6\times6$ resistant matrix as below;

\[

\begin{bmatrix} F\\

T \end{bmatrix} =\begin{bmatrix} A & B \\

C & D \end{bmatrix} \begin{bmatrix} \nu

\\ \omega

\end{bmatrix}

\]

Where $A$, $B$ and $D$ are matrices 3x3 and only depend on the object\rq{}s geometry and fluid velocity.

In the study performed by \citeauthor{purcell1997efficiency} \citep{purcell1997efficiency} it has been proved

matrices $B$ and $C$ are equal ($B = C$) for a typical flagellum.

%\begin{figure}

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\begin{wrapfigure}{r}{0.5\textwidth}

\begin{center}

\includegraphics[width=0.5\textwidth]{nanoJet3}

\caption[Drilling motion of a nanotube]{Demonstrating the drilling motion of the nanotubes under rotating

magnetic field~\citep{C2NR32798H}.}

\label{nanotube}

\end{center}

\end{wrapfigure}

%\end{figure}

There are few methods in use to model the resistance matrices and low Reynolds flow such as the

method of regularized stokeslets, the boundary element method and the method of fundamental solution

. In designing a microrobot the main parameters required to concentrate on are the helicity angle $\psi$,

the helix radius $R$, the pitch $p$ and the filament radius $r$ as illustrated in Figure~\ref{ref8} part (c).

\begin{figure}

\centering

\includegraphics[width=0.7\textwidth]{8}

\caption[The prototype of microhelical device]{ The prototype of microhelical device. (a) Scanning electron microscopic image of the micro polymer composite

with the 2 vol.\% nanoparticle fill factor and (b) 4 vol.\% of nanoparticle fill factor. (c) The CAD model

shows all the parameters required for the microhelical design ~\citep{peyer2013bacteria}.}

\label{ref8}

\end{figure}

Magnetic actuated microrobot is divided into two categories; torque driven microrobot and force

driven microrobots.

The micro robot using the torque-driven method is more favourable than the force-driven method

because their rotation is based on applying torque rather than a force to pull the device ~\citep{peyer2013bacteria}.

Another approach for powering a micro robot is using the catalytic conversion of chemical energy

into mechanical energy (Figure~\ref{nanotube}). In this method, the catalyst accelerates the consumption of hydrogen peroxide

and helps the self-propulsion of micro robot to pump the fluid to transport cells and colloidal

particles ~\citep{C2NR32798H}. The catalytic tube is fabricated with a sub micrometer diameter.

This technique is not applicable for the minimally invasive surgery (MIS) yet because the catalytic

material used in the fabrication process of nanotubes is toxic. Hence, biocompatible fuel is required to be developed in order to

apply this technique in a live cell environment~\citep{C2NR32798H}.

Alternatively, the micro driller can be powered and controlled by using an external magnetic field

such that changes in the frequency of the rotating magnetic field switch the rotational orientation of the

micro tool from the horizontal position to the vertical one. The vertical orientation of the rolled up microtube

and its sharp helical design makes the device suitable for drilling into biological tissue. In addition, the micro

driller can be used for targeted drug delivery in MIS ~\citep{C2NR32798H}.

\subsubsection{Plant-based microrobots}

%\hilight{details about fabrication, extract xylem tissue}

The helical microstructures are not limited to having flagellum-like structures and microbots with

general cilia-like feature have been designed. \citeauthor{gao2013bioinspired}

observed the helical microstructures that imitates spiral water-conducting vessels of different plants.

\begin{figure}

\centering

\includegraphics[width=0.9\textwidth]{plants}

\caption[Xylem\rq{}s shape in different plants ]{The shape of the Xylem in differnt plants~\citep{mahoney2011velocity}.}

\label{plants}

\end{figure}

In order to obtain unstretched spiral vessel several plants were collected and their leaves were

macerated and washed with pure water. Tweezers were used to uncover compressed spiral vessels

in the planar networks. Leaves were gently scored and two segments were pulled apart to a permanent

length to stretch the spiral vessels. These spiral vessel were kept in a glass slides and covered with a

thin layer ($20 nm$) of titanium and nickel ($80 nm$) using an

E-beam evaporator ~\citep{mahoney2011velocity}. The helical vessels were coated in nail

polish and baked for 2 minutes to impound the helix and protect the structure. The final product is

a photoresist film on glass that was cut into required lengths.

The fabrication process involves coating isolated spiral xylem vessel plant fibres within a (Figure~\ref{ref8})

thin magnetic layer. Xylem tissue transports the plant\rq{}s required food such as water and other

nutrition from the root to the leaves using capillary action ~\citep{mahoney2011velocity}.

Use of plant material in this method enables simple three-dimensional microswimmers fabrication

and biocompatibility. In addition, the magnetic cover helps to ensure accurate directional control and

high-speed propulsion. Therefore, the fabrication processes were extremely simplified as the main

component of the helical microswimmers is from nature and more than a million individual micro helicals

can be made from a very small section of the plant stalk ~\citep{mahoney2011velocity}. Using mechanical stretching can control geometric variables of the helical vessels such as the pitch and

helix angle and hence plenty of helical microswimmers can be reproduced. The final shape of the

helical microswimmer is determined mainly by the initial diameter of the unstretched spiral vessel.

The process of stretching helical plant structure was performed via plastic deformation so that the number

of helical turns are constant and tensile stretching of the plant fibre stretching is negligible~\citep{mahoney2011velocity}.

The method used for precise propulsion control and characterising the locomotion behaviour of the

plant-based microswimmers is similar to the method applied in \citeauthor{gao2013bioinspired} study.

According to \citeauthor{gao2013bioinspired} ~\citep{gao2013bioinspired} experiment, the plant-based

microswimmers exhibited high speed movement ($85~\mu m$) in raw biological medium such as

pure human serum under the rotating magnetic field. However, their swimming speed in pure water

($90~\mu m$) was slightly higher than human serum.

%\begin{figure}

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\begin{center}

\includegraphics[width=0.5\textwidth]{plants2}

\caption[plant-based microrobot]{(A) The stages were required to make a plant-based microrobot. (B) A microscopic image of the

a xylem helical structure~\citep{gao2013bioinspired}.}

\label{plants2}

\end{center}

\end{wrapfigure}

%\end{figure}

Hence, an increased velocity of the

biological fluid has a minor effect on the plant-driven microswimmers, which is an important

advantage of this microdevice over the common microrobots%\hilight{WHY}.

\subsection{Actuation methods} \label{actuation}

The actuation method for swimming microrobot should meet two main criteria in order to be

applicable. The method needs to be appropriate in the fluid environment and can be applied

in the micro scale. One approach was using tethered and onboard motor to an external power

source to actuate the microrswimmers, but this approach will not be realistic in micro scale.

Therefore, using the propulsion mechanism of natural swimmers such as flagellum demonstrated

a successful result~\citep{peyer2013bio}.

Another approach is using electrochemical decomposition for microrobot locomotion.

The mechanism of these types of artificial microdevice is similar to bacteria as both harvest the

required energy from their environment. In that case, the environment contains chemical material such

as hydrogen peroxide to make the electrochemical reaction. The successful application of these catalyst

microdevice in vitro is reported for cell transportation. However, this approach will not be suitable for in vivo

cases where the chemical material may harm the human body~\citep{peyer2013bio}.

Therefore, the idea of using magnetic field for the microrobot actuation satisfied both requirements.

Applying the low strength magnetic field is harmless for the human body and that can be used within fluidic

environment. So it is possible to have microswimmers in the fluid environment and to control them remotely. However,

there are still challenges with using a magnetic field as an actuation method. The magnetic field will decay

fast by increasing the distance from the magnetic source. Thus, that factor needed to be considered

when preparing the set up for actuated microrobot~\citep{peyer2013bio}.

The microrobot actuation by magnetic field can be force driven or torque driven. In the case of the torque

driven, the magnetized microrobot experiences a torque that perform to align its magnetization with the

external magnetic field. The magnetic torque and force are formulated as follow;

\begin{equation}

\bm{T}\_m = V\bm{M} \times \bm{B}

\label{originalForce}

\end{equation}

\begin{equation}

\bm{F}\_m = V(\bm{M\nabla}) \bm{B}

\label{originalTorque}

\end{equation}

Where $\bm{T}\_m [N.m]$ is torque, $\bm{F}\_m [N]$ is force, $\bm{M} [A.m^{-1}]$ is magnetization, $V [m^3]$ is

volume of a magnetized object and $\bm{B} [T]$ is the magnetic field. If we have a hard magnet, $\bm{M}$ becomes

a constant or it can be a function of the geometry of the object and applied field. In the uniform magnetic field,

there is no force and microrobot just experiences the torque until the magnetization $\bm{M}$ is collinear with the

magnetic field. At this point, there is no torque and the microswimmer remains stationary. Thus, a magnetic field is required

to go through spatial or temporal changes to generate a continous actuation. This can be performed by

rotating the helmholtz coils or generaring a dynamic magnetic field by using AC current~\citep{peyer2013bio}.

\subsection{Fabrication methods} \label{fabrication}

Historically, the fabrication of the microrobot was the main problem to be resolved, but micro-scale fabrication methods

offer a feasible solution~\citep{gao2013bioinspired}.

In 2007, the first artificial bacteria flagella was fabricated based on thin-film deposition and self-scrolling

methods~\citep{qiu2014noncytotoxic}. They used InGaAs/GaAs bilayer for fabricating

helical tail and Ni for actuation microrobot\rq{}s head. The similar fabrication method employed by Zhang

in 2009 with the addition of a Cr layer between the microrobots\rq{} tail and its head~\citep{qiu2014noncytotoxic}.

An improved adhesion of microrobot was the result of adding Cr layer.

3D laser direct writing (DLW) and electron beam decomposition are methods used since then. A typical

fabrication process consists of two stages. Initially, the core structure of the artificial helical

microswimmer is printed using 3D lithography, following which electron beam evaporation is used for

ferromagnetic thin film coating~\citep{tottori2013artificial}.

Performance of each microswimmer (with different design) can be imaged by the scanning electron

microscope (SEM). After the fabrication process is completed, the next step is to release the structure into

deionised water using the tungsten probe. The tank with deionised water is installed in the middle of the

three-axis Helmholtz setup.

\begin{figure}

\centering

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\includegraphics[width=0.7\textwidth]{tempreture}

\caption[Direct lasor writing]{DLW steps. step 1 is writting helical microrobots, in step 2 microrobots were

developed in isopropyl alcohol and step 3 is coating them by a layer of Ni and Ti. ~\citep{qiu2014artificial}.}

\label{tempreture}

%\end{center}

%\end{wrapfigure}

\end{figure}

To improve biocompatibility for in-vivo applications, the

microrobot can be covered with a thin layer of titanium. In addition, the microrobot\rq{}s structure was layered with

nickel for the purpose of magnetic actuation.

\citeauthor{qiu2014artificial}~\citep{qiu2014artificial} reported a successful application of helical microrobots

for drug delivery were known as \lq\lq{}smart\rq\rq{} drug carriers. Again, they used DLW for the fabrication method

as shown in the Figure \ref{tempreture}.

The smart drug carriers were coated in a layer of temperature-sensitive liposomes which is composed

of a lipid bilayer and was proposed for cancer therapy in local hyperthermia treatments~\citep{qiu2014artificial}.

The main component of temperature-sensitive liposomes is Dipalmi- toylphosphatidylcholine (DPPC) which

transforms from solid to liquid gel at the $41^{\circ} C$ and released encapsulated drugs.

\paragraph{}

\citeauthor{qiu2014noncytotoxic}~\citep{qiu2014noncytotoxic} used commercially available material such as ORMOCOMP

for fabrication of helical microrobots in their recent experiment. ORMOCOMP is a biocompatible photoresist which

can improve the potential use of microrobots for in vivo applications becuase it supports viability, cell proliferation

and normal morphology of various cell lines. For the purpose of magnetic microrobot actuation, soft magnetic material

such as Fe, Ni and Co are commonly used in microscale structures. The main reason is their biocompatibility

with surface decomposition methods, however Ni and Co are cytotoxic and pure iron can be biodegradable~\citep{qiu2014noncytotoxic}.

ORMOCOMP helical swimmers were coated onto a thin layer of Fe ($25 nm$) using electron beam decomposition.

\begin{comment}

The review of all the fabrication methods used for the micrordevice is represented in the table\ref{fabrication\_table}.

\begin{table}[!ht]

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}

\label{fabrication\_table}% is used to refer this table in the text

\caption{Microrobot\rq{}s fabrication methods}\label{fabrication table}% title of Table

\end{table}

\end{comment}

%%%%%%%%%%%%%%% Table %%%%%%%%%%%%%%

\begin{table}[h!]

\centering

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\setlength{\tabcolsep}{1pt}

\renewcommand{\arraystretch}{2.8}

\begin{tabular}{ c m{2.5cm} m{4.3cm} m{3cm} m{2cm}}

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Microrobot Image & Design & Fabrication Method & propulsion method &Citation \\ \hline\hline

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\item Helical Screw Shape

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\item Direct Laser Writing (DLW)

\item Two-photon Polymerization

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\item RFT

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\item \citep{peyer2013magnetic}

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\item Helical rigid tail

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\item Direct Laser Writting (DLW)

\item Two-photon Polymerization

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\item RFT

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\item \citep{peyer2013bio}

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%%%%%%%%%%%%%%%% Third row

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\includegraphics[width=\linewidth, height=25mm]{Planar\_flexible\_tail}

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\item Planar flexible tail

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\item The EMA coil system

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\item SBT

\item RFT

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\item \citep{kim2013fabrication}

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\item Cilia

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\item The EMA coil system

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\item \citep{kim2013fabrication}

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\item Nanotube

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\item Molecular Beam Epitaxy (MBE)

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\item Drilling motion

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\item \citep{C2NR32798H}

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\begin{minipage}{.3\textwidth}

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\item Plant-based

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\begin{itemize}

\item Macerating Plant\rq{}s Leaves

\item Seperating Spiral Vessels

\item Stretching spiral Vessels

\item Coating with Ti

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\begin{itemize}

\item RFT

\end{itemize}

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\begin{itemize}

\item \citep{gao2013bioinspired}

\end{itemize}

\\ \hline

\end{tabular}

\caption[Summery of microrobots\rq{}s desing, locomotion and fabrication]{Different types

of microrobots, their fabrication and propulsion method.}\label{Micro}

\end{table}

%%%%%%%%%

%%%%%%%%%%%%%%%%%%%% Method %%%%%%%%%%%%%

%%%%%%%%%

\chapter{Methods}

In this chapter the design of microhelix is described and a few number of design is presented. It followed by introducing

the fabrication mechanism, the technology applied to fabricate microstructures and post-processing fabricated microstructres.

The simulation is major part of this chapter which is modelling the swimming mechanism of microhelix in high viscous fluid.

Three models studied for describing the swimming motion of the microhelix and one model (\ac\*{RFT}) is implemented to simulate

microhelix with two and six degrees of freedom. The torque driven magnetic field is selected to actuate the microhelix and

as a result the relation between the rotational velocity and translational velocity is achieved.

%%%%%%%%%%%%%%%%%%% Design %%%%%%%%%%%%%%%

\section{Microrobot design} \label{microDesign}

For the purpose of this study, the design of microrobots is focused on a helical

tail shape with possible propeller head. The helical shape microrobot has generally copied the design of the helical

rigid tail flagellum which is a one-dimensional structure. There are other microstructures

such as cilia and planar flexible tail flagellum that are copied to build a swimming microrobot. The

helical rigid tail flagellum is a preferred design for the micro swimmer as its simplicity makes

it feasible to copy in micro scale. In addition, its swimming mechanism is more efficient than

other type of microswimmers \citep{peyer2013bio}.

Therefore, the key characters of the helix were

identified and all the design was based on optimising these characters.

\begin{figure}

\centering

\begin{subfigure}[b]{0.25\textwidth}

\includegraphics[width=\textwidth]{Design6}

\caption{Circle base helix with sphere head}

\label{Design6}

\end{subfigure}~

\begin{subfigure}[b]{0.36\textwidth}

\includegraphics[width=\textwidth]{Design2}

\caption{Rectangle base helix}

\label{Design2}

\end{subfigure}~

\begin{subfigure}[b]{0.32\textwidth}

\includegraphics[width=\textwidth]{Design3}

\caption{Rectangle base helix}

\label{Design3}

\end{subfigure}

\begin{subfigure}[b]{0.335\textwidth}

\includegraphics[width=\textwidth]{Design4}

\caption{Rectangle base helix with sphere head}

\label{Design4}

\end{subfigure}~

\begin{subfigure}[b]{0.37\textwidth}

\includegraphics[width=\textwidth]{Design5}

\caption{Rectangle base helix with square head }

\label{Design5}

\end{subfigure}~

\begin{subfigure}[b]{0.222\textwidth}

\includegraphics[width=\textwidth]{Design1}

\caption{Circle base helix}

\label{Design1}

\end{subfigure}

\caption[Different helical designs]{(a) Circle base filament helix with a sphere

head and three turns. (b) The rectangle base filament helix with larger side of the rectangle

was revolved about a spiral path (b) whilst in (c) smaller side of the rectangle revolved

about a spiral path. The rectangle base helix can be integrated to sphere head (d) or square

head (e).}\label{Different helical designs}

%add desired spacing between images, e. g. ~, \quad, \qquad, \hfill etc.

%(or a blank line to force the subfigure onto a new line)

%add desired spacing between images, e. g. ~, \quad, \qquad, \hfill etc.

%(or a blank line to force the subfigure onto a new line)

\end{figure}

The figure \ref{Different helical designs} represents the

variaty of designs that have been undertaken in the design stage of this project. A circle base helix with (\ref{Design6})

and without sphere head (\ref{Design1}) has been designed. Also their pitch, helical angle and length of the helix

were changed to monitor its behaviour during simulation and fabrication process. Two types of

rectangle base helix has designed. In the first one, the larger side of the rectangle revolves around a spiral path (\ref{Design2})

whilst in the second one the smaller side of the rectangle revolves around the spiral path (\ref{Design3}).

The former design advantages in the simulation process because its shape produce larger force to propell the microrswimmer.

However, the it behave poorly in the fabrication process because it has not provide sufficient surface area to contact with the

substrate. Therefore we can\rq{}t print it vertically. The latter provides strong base for printing vertically but ca\rq{}t generate

sufficient force to move the microswimmer forward. The new design of microhelix which has

variable pitchrather than constant pitch is made (\ref{Design7}) and fabricated vertically. The advantage of variable pitch design

is providing strong base to fabricate it vertically.

%%%%%%%%%%%%%%%%%% Fabrication %%%%%%%%%%%%%

\section{Microrobot fabrication} \label{microFabric}

The main challenge of the fabrication process is not just fabricating the extremely small object.

There are number of other factors that need to be considered to select an appropriate approach for

fabricating helical shape mincrorobot. An ideal fabrication approach should have control over design

parameters and in particular it should be suitable for applying magnetic material

for the actuation purpose \citep{peyer2013bio}.

%\begin{figure}

% \centering

\begin{wrapfigure}{r}{0.4\textwidth}

\begin{center}

\includegraphics[width=0.3\textwidth]{Design7}

\caption[Variable pitch helix design]{Variable pitch helix design}

\label{Variable pitch helix design}

\end{center}

\end{wrapfigure}

%\end{figure}

\begin{figure}

\centering

%\begin{wrapfigure}{r}{0.5\textwidth}

%\begin{center}

\includegraphics[width=1.0\textwidth]{Fabrication\_diag}

\caption[Fabrication process overview]{Fabrication process overview. The last block

of the diagram on the right (dark gray) did not try in this study. }

\label{Fabrication\_diag}

%\end{center}

%\end{wrapfigure}

\end{figure}

The complete fabrication process is summarised in the diagram \ref{Fabrication diagram}.

The process started by importing the structure files into the software called Describe for

the pre-processing purpose. The structures file contains all the microrobots that is designed by the

software called Solidwork. The next stage is writing the structure using nanoscribe facility, which is

carried out in the clean room. After the structures printed, they can be seen under

the \ac\*{SEM}. However, the structure made of polymer and the pictures of non-metal object under

the \ac\*{SEM} is not perfect. Thus the polymer made microrobot were first coated with a layer of

metal (usually gold) and this process is called sputtering coating process. At this stage, the

picture of microrobot were analysed under the \ac\*{SEM} and if they are satisfactory, they will go to

the final stage which is magnetization process. However, we the magnetization stage was not attempted

in this project because the aim was to optimise the structures in terms of design and fabrication and

there was no plan to run an experiment within the short period of time. The pictures identified

as having a problem were sent to the design stage for further optimisation process. In the following

two sections, we will explain the mechanism of the two photon lithography techniques and \ac\*{SEM} in

more detail.

%%%%%%%%%%%% Overview Diagram of the fabrication process %%%%%%%%%%%%%

\begin{comment}

\begin{figure}

\centering

\begin{tikzpicture}[scale=1.10,transform shape]

% Draw diagram elements

\path \Step{1}{Preparing};

\path (p1.south)+(0.0,-1.5) \Step{2}{Production};

\path (p2.south)+(0.0,-1.5) \Step{3}{Post-processing};

\path (p3.east)+(3.0,0.0) \Step{4}{Sputter Coating};

\path (p4.south)+(0.0,-1.50) \Step{5}{Image Analysis};

\path (p5.east)+(3.0,0.0) \Step{6}{Coating Magnetic Material};

%\path (p4.east)+(5.0,0.0) \practica{7}{Calculate forward and inverse kinematic};

%\path (p5.east)+(5.0,0.0) \practica{8}{Design feedback control mechanics};

%\path (p5.south)+(3.5,-2.0) \practica{9}{Integrating e-AR and robot};

%\path (p9.south)+(0.0,-1.5) \practica{10}{Laparoscopy Robot};

% Draw arrows between elements

\path [line] (p1.south) -- node [above] {} (p2);

\path [line] (p2.south) -- node [above] {} (p3);

\path [line] (p3.east) -- node [above] {} (p4);

\path [line] (p4.south) -- node [above] {} (p5);

\path [line] (p5.east) -- node [above] {} (p6);

% \path [line] (p7.south) -- node [above] {} (p8);

%\path [line] (p5.south) -- node [above] {} (p9);

%\path [line] (p8.south) -- node [above] {} (p9);

% \path [line] (p9.south) -- node [above] {} (p10);

\background{p1}{p1}{p3}{p3}{Nanoscribe }

\background{p4}{p4}{p5}{p5}{SEM}

% \path [line] (p5.south) -- node [above] {} (bk3-n);

% \path [line] (bk3-s) -- node [above] {} (p8);

% \path [line] (bk3-s) -- node [above] {} (p9);

%\path (bk1-e)+(+6.0,0) node (ur1)[ur] {};

% \path (bk2-w)+(+6.0,0) node (ur2)[ur] {};

%\path (bk3-w)+(+3.0,0) node (ur3)[ur] {};

% \transreceptor{bk1-e}{pre processing}{ur1};

% \transreceptor{bk2-w}{Feature selection}{ur2};

%\transreceptor{bk3-w}{classification}{ur3};

%%%%%%%%%%%%%% CHANGED%%%%%%%%%%%%%

\end{tikzpicture} %

\caption[Fabrication overview]{Fabrication diagram. The nanoscribe technology is employed

for the fabrication process and it was followed by analysing structure under SEM.

The final stage (yellow block in third column) has not been attempted in this project. } %

\label{Fabrication diagram} %

\end{figure} %

%

%%%%%%%%%%%%%% CHANGED%%%%%%%%%%%%%

\end{comment}

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

\subsection{Nanoscribe}\label{nanoscribe}

Nanoscribe is a company providing a sophisticated system and device for true 3D

micro and nanofabrication. Their system is based on the laser lithography and it

used two-photo polymerization technique for the fabrication. The fabrication device combines

two modes for writing; the high-speed galvo-mode and an ultra-precise piezo-mode. The former

is for fastest fabrication and it makes the structure in a layer-by-layer process. The latter is mainly for

printing arbitrary 3D trajectories~\citep{Doe:2014Feb:Online}. The complete nanoscribe

package is made up of three components, Photonic Professional GT, the software and IP Photoresists.

Photosensitive material is used in both modes for structuring arbitrary 3D patterns in a high-resolution.

The properties of the photosensitive material, the laser power and the size of the spot in the material

determined the voxel size. Extremely small voxel size can be achieved when focusing optics is used with

a high numerical aperture. The fabrication process in each mode is based on moving the voxel relative to

the sample. The galvo mode approach is called \ac\*{MBFS} in which the laser beam is scanned

and galvanometric mirrors and piezo-actuators will control the vertical movement. However in

piezo-mode, piezo actuators move the substrate in all three dimensions to achieve a highly

precise focus trajectory. This type of implementation is know as \ac\*{FBMS}~\citep{Doe:2014Feb:Online}.

\begin{figure}

\centering

\includegraphics[width=0.9\textwidth]{nanoscribeWorkflow}

\caption[Nanoscribe workflow]{nanoscribe workflow. The stages are involved in 3D micro-printing with nanoscribe

device is shown in the diagram~\citep{Doe:2014Feb:Online}.}

\label{nanoscribeWorkflow}

\end{figure}

\paragraph{}

The whole process of fabrication is formed of three stages; preparation, production and post-processing.

In the first stage, the \ac\*{stl} file that contains the design of structures will import into the software tool

called DeScribe. In this software, each design will go though three steps for fixing the mesh, slicing and

hatching. By completing these three steps, the result will be the \ac\*{gwl} format file that is ready for the

production stage.

The production is the stage for controlling the print job that has been done by user-friendly

graphical interface software, NanoWrite. This software controls different aspect of the lithography

system such as autofocus, exposure dose and substrate positioning.

The final stage involves removing the excess resin in order to improve the visual quality~\citep{Doe:2014Feb:Online}.

%\begin{figure}

% \centering

\begin{wrapfigure}{r}{0.5\textwidth}

\begin{center}

\includegraphics[width=0.5\textwidth]{SEM}

\caption[Scanning Electron Microscope]{Scanning Electron Microscop. The components of the \ac\*{SEM} is shown

in the diagram~\citep{Doe:2014Jan:Online}.}

\label{SEM}

\end{center}

\end{wrapfigure}

%\end{figure}

IP Photoresists is a high viscose fluid that comes as part of nanoscribe package. It is

used to maximise the performance of the multiphoton polymerization process. It has high

mechanical stability and sticks very well to different substrate ~\citep{Doe:2014Feb:Online}. A wide

range of material with different mechanical, optical or chemical properties can be used for the substrate

in direct laser writing. The choice of the substrate material is application dependent. For example in

optical applications the transparent material such as glass is more appropriate. In the latter application, the

substrate is mainly for supporting to the polymer structures. Also, we can use pre-structured substrates

such as transparent micro-fluidic chip so that polymer structure can be printed on the substrate. In that

case, the functionality of multiphoton lithography can be improved by combining the mechanical parts

with substarte~\citep{Doe:2014Feb:Online}.

\paragraph{}

In this project we used a nanoscribe device to print microswimmers using the galvo-mode of the machine.

The whole printing unit is located in the clean room\footnote{Clean room is an environment with controlled

concentration of airborne particles to make it suitable for product manufacturing~\citep{Doe:2014April:Online}. }.

The fabricated structures were observed under the

\ac\*{SEM} and all the resulting images are presented and discussed in the result \ref{result} and discussion

chapters \ref{discussion} respectively.

\subsection{Scanning Electron Microscope (\ac\*{SEM})}\label{sem}

\ac\*{SEM} is a powerful device for obtaining high magnification images to analyse and examine

the material or individual features. \ac\*{SEM} was invented 50 years ago and is used extensively

in diverse scientific fields such as biology, medicine or metallurgy, to name just a few. The \ac\*{SEM} can

provide images with the high-resolution down to $25$ Angstroms.\footnote{$1 ~\ Angstrom =

1.0 \times 10^{-10} Metres$}%~\citep{Doe:2013Dec:Online}.

\ac\*{SEM} generates a range of signals at the surface of solid specimens by using a focused beam

of high-energy electrons. The process starts with the electron gun producing an electron beam

at the top of the microscope which then travels into the microscope. The microscope is placed in the

vacuum. The beam is then focused down onto the specimen by passing through the electromagnetic

fields and lenses. Once the focused electron beam interacts with the specimen, electrons are revealed

from the specimen. At this point, the back-scattered electrons are collected by the detectors and

converted into variety of signals. Ultimately, generated signals sent to the screen to form the final image

of the specimen\citep{Doe:2014Jan:Online}.

The key advantages of using \ac\*{SEM} over traditional microscope is having the large depth of

field and the higher resolution. In addition, researchers have more control over the degree of

magnification because \ac\*{SEM} uses electromagnets\citep{Doe:2014Jan:Online}.

We need to prepare samples before using the \ac{SEM} because it uses the electron in a

vacuum condition. That means the sample should not contain any water as otherwise the water will

vaporise in the vacuum. This is a high-vacuum \ac\*{SEM}. If we require an image of a wet sample such

as biological specimen we can use the low-vacuum \ac\*{SEM}. In that case, the specimen chamber

contains air that avoid dehydrating of samples.

Because the produced images are based on the electron-sample interaction, if the sample is made

of non-metal material, the final image is not very clear. Thus, the sample needs to be covered by a metal in order to

make it conductive. The process of covering the sample with metal is called sputter coating\citep{Doe:2014Jan:Online}.

\paragraph{sputter coating}

In the sputter coater, there is small chamber in the vacuum to place the sample in. An electric field

and argon gas is used in order to release the electron from the argon and convert it into positively charged

atom. Then, argon ions and negatively charged gold foil are attracted to each other and as a result gold

atoms fall from the surface of the gold and settle onto the surface of the specimen. Therefore, a thin gold

layer covers the surface of the sample and makes it conductive for \ac\*{SEM} machine\citep{Doe:2014Jan:Online}.

In this project, the microrobot structure is made of polymer, which we conductive by applying the

sputter coating process and then analysing them under \ac\*{SEM}. The images of the microrobot structures

before and after coating is presented in the result section \ref{result}. In the following section, the simulation of

microrbot is demonstrated in detail.

%%%%%%%%%%%%%%%%%% Simulation %%%%%%%%%%%%%

\section{Simulation}\label{simulation}

\begin{figure}

\centering

%\begin{wrapfigure}{r}{0.5\textwidth}

%\begin{center}

\includegraphics[width=1.0\textwidth]{Simulation\_Diag}

\caption[Fabrication process overview]{Simulation workflow. The microrobot

desired velocity is an input of a system. The electric current is required to generate a magnetic field for microrobot

actuation is an output.}

\label{Simulation\_Diag}

%\end{center}

%\end{wrapfigure}

\end{figure}

The simulation of the microrobot is formed by two main components; helical microrobot propulsion mechanism

and actuation method. The complete algorithm that describes the implementation of the simulation

system is represented in the diagram \ref{PSuedocode}. In this algorithm, the desired velocity is given to the system

and the required electric current to make the dynamic magnetic field will be an output of the system. The pseudocode of

the algorithm \ref{PSuedocode} is provided in more details in each step of the implementation and the complete computation

is explained in section \ref{RFT\_sixDegree} and section \ref{microActuation}.

\begin{comment}

\begin{figure}

\centering

\begin{tikzpicture}[scale=1.30,[every node/.style={font=\large,

minimum height=1.5cm,minimum width=0.8cm},]]

% Draw diagram elements

\path \Step{1}{Desired Velocity};

\path (p1.east)+(3.0,0.0) \Step{2}{Propulsion Algorithm};

\path (p2.east)+(3.0,0.0) \Step{3}{Actuation Algorithm};

\path (p3.south)+(0.0,-2.0) \Step{4}{Torque Calculation};

\path (p4.west)+(-3.0,0.0) \Step{5}{Magnetic Field Calculation};

\path (p5.west)+(-3.0,0.0) \Step{6}{Electric Current};

% Draw arrows between elements

\path [line] (p1.east) -- node [above] {} (p2);

\path [line] (p2.east) -- node [above] {} (p3);

\path [line] (p3.south) -- node [above] {} (p4);

\path [line] (p4.west) -- node [above] {} (p5);

\path [line] (p5.west) -- node [above] {} (p6);

% \background{p3}{p3}{p5}{p5}{Simulation }

%%%%%%%%%%%%%% CHANGED%%%%%%%%%%%%%

\end{tikzpicture} %

\caption[Simulation algorithm]{Simulation algorithm. The main components of implementing the simulation is shown in

the diagram.}

\label{Simulation algorithm} %

\end{figure} %

%

%%%%%%%%%%%%%% CHANGED%%%%%%%%%%%%%

\end{comment}

%%%%%%%%%%%%%%%%%%%Pseudocode%%%%%%%%%%%%%

\begin{algorithm}[H]

\KwData{Velocity ($\bm{V}$), RFT, RSM, SBT}

%\KwResult{how to write algorithm with \LaTeX2e }

%initialization\;

\While{ $\bm{V} \neq 0$}{

Select propulsion method from (RFT, RSM, SBT)\;

Compute propulsion matrix coefficient $(b,c)$\;

Decompose $\bm{V} $ to ${ \bm {{V}\_{hor}}}$ and ${ \bm {{V}\_{ver}}}$\;

\eIf{${\| \bm {{V}\_{hor}}\|} = 0$}{

Rotational velocity $ \bm {\Omega} = \frac{{\| \bm{V}\|}+ d\_{11}\| \bm{f}\|}{e\_{11}}$\;

Microrobot direction point $\tilde{\bm{X}} = -\hat{\bm{g}}$\;

Go to next step

}{

Rotational velocity $ \Omega = \frac{\| {\tilde{\bm{V}} \| \cos(\psi) } + d\_{11} \| \bm{f} \| \cos(\psi - \alpha)}{e\_{11}}$\;

Microrobot direction point $\tilde{\bm{X} } = \frac{\tilde{\bm{V}}}{\| {\tilde{\bm{V}}} \|}$\;

Go to next step\;

}

Compute Torque $\tau = b \bm{V} + \Omega c$\;

Compute Magnetic field $\bm{B}$ from $\tau = \bm{V}M \times \bm{B}$\;

Compute Electric current $i$ from $|\bm{B}| = (\frac{b^2}{(b^2+l^2)^{3/2}}){\mu}\_0 i$

}

\caption[Simulation details]{Simulation algorithm}

\label{PSuedocode}

\end{algorithm}

%%%%%%%%%%%%%%%%%%%END of Pseudocode%%%%%%%%%%%%%

\subsection{Modelling helical propulsion}\label{maths}

Analysing fluid dynamic phenomena on microorganism is a fundamental approach to model

microorganism

locomotion~\citep{smith2009boundary}.

A helical bacterial flagellum can be uesed as a reference to model a helical microrobot. The

essential parameters to model a helix are, helix length ($L$), pitch ($\lambda$), pitch angle ($\theta$),

radius ($R$), filament radius ($a$) and contour length ($\Lambda = L/ \cos \theta$). Figure~\ref{parameters} shows

the helix parameters evidently~\citep{rodenborn2013propulsion}. The flagella parameters were measured for

several species of bacteria and its result showed the helical pitch is typically ranging between $2R$ and

$11R$, ($2R < \lambda < 11R$). Also the helix length ($L$) varies from $3\lambda$ to $11\lambda$,

($3\lambda < L < 11\lambda$).

\begin{figure}

\centering

%\begin{wrapfigure}{r}{0.5\textwidth}

%\begin{center}

\includegraphics[width=0.7\textwidth]{parameters}

\caption[Helix parameters]{The essential helix parameters to design a helical microrobot~\citep{rodenborn2013propulsion}.}

\label{parameters}

%\end{center}

%\end{wrapfigure}

\end{figure}

The flagella rotation at low Reynolds number exerts an axial thrust ($F$) and torque ($T$) related to the

rotation rate ($\omega$) and flagellum axial velocity ($\nu$). At the same time, fluids was exerted on the force

($-F$) and the torque ($-T$) on the swimming microrobots~\citep{purcell1997efficiency}. The fluid dynamic

is governed by the Stokes equations (\ref{stokes\_1}) in the low Reynolds regime;

\begin{equation}

-\nabla{p}+ \eta\nabla^2{\nu} = 0

\label{stokes\_1}

\end{equation}

Where $\eta$ and $p$ are fluid dynamic velocity and pressure respectively. Therefore, thrust ($F$) and

torque ($T$) are linearly related to the $\nu$ and $\omega$ as there is no derivation of time in

the equations \ref{stokes\_1}. These linear relationship can be defined as follow;

\begin{equation}

F = A\nu + B\omega

\label{linear1}

\end{equation}

\begin{equation}

T = C\nu + D\omega

\label{linear2}

\end{equation}

Therefore, a matrix

$\bigl(\begin{smallmatrix}

A&B\\ C&D

\end{smallmatrix} \bigr)$

defined as propulsion matrix the

model to explain the flagella swimming motion described by following equation~\citep{rodenborn2013propulsion}

as mentioned in the literature review earlier;

\[

\begin{bmatrix} F\\

T\end{bmatrix} = \begin{bmatrix} A & B \\

C & D \end{bmatrix} \begin{bmatrix} \nu

\\ \omega

\end{bmatrix}

\]

The elements in the symetric $2\times2$ matrix (propulsive matrix) in the above equation only depends on

flagellum geometry. The propulsive matrix elements can be computed by three methods called;

resistive force theory, slender body theory and regularized Stokeslet theory which are described in detail in sections \ref{method3},

\ref{method1} and \ref{method2} respectively.

\subsubsection{Resistive force theory for microrobot with two degrees of freedom}\label{method3}

\begin{figure}

\centering

%\begin{wrapfigure}{r}{0.5\textwidth}

%begin{center}

\includegraphics[width=0.8\textwidth]{motion}

\caption[Microrobot filament motion]{Analysis of arbitrary filament motion of microhelix~\citep{edd2003biomimetic}.}

\label{motion}

%\end{center}

%\end{wrapfigure}

\end{figure}

The swimming velocity and efficiency of the microrobot can be predicted by Resistive force theory (RFT)~\citep{purcell1997efficiency}.

The force exerted on the fluid by micro swimmer were calculated initially and the micro swimmer will have a net

movement if the force is not zero~\citep{Doe:2013:Online}. Furthermore, the swimming velocity will decrease if the helical

body is attached to the innert head. Figure \ref{filament} shows an arbitary filament motion which is defined by $s(l,t)$.

A direction of the helix velocity ($U$) is along x-axis and its rotation is symmetric about the x-axis.

The following assumption has been made in order to use the RFT.

%\begin{figure}

% \centering

\begin{wrapfigure}{r}{0.5\textwidth}

\begin{center}

\includegraphics[width=0.5\textwidth]{filament}

\caption[A motion of helical segment]{A motion of an arbitrary filament~\citep{edd2003biomimetic}.}

\label{filament}

\end{center}

\end{wrapfigure}

%\end{figure}

The geometry

of the helix is on the yz-plane and it always attached to the robot body (can be a sphere). The

filament motion is periodic and the filament length is constant at all the time. Acceleration can be neglected as the

system is in the low Re number fluid. Hence, the equations \ref{thrust\_1} and \ref{torque\_1} will describe the force balance and

the moment balance in the x-axis direction. The thrust and torque will be determined by integrating over the

first term of the force balance and moment balance equations (\ref{thrust\_1} and \ref{torque\_1})

respectively ~\citep{edd2003biomimetic}.

\begin{equation}

\cfrac {1} {\Delta T} \int\_0^{\Delta T} \int\_0^L f\_x(l,t)\: \mathrm{d}l \:\mathrm{d}t + C\_DU = 0

\label{thrust\_1}

\end{equation}

\begin{equation}

\cfrac {1} {\Delta T} \int\_0^{\Delta T} \int\_0^L [\bold r \bold \times \bold f(l,t)] . \bold e\_1x \: \mathrm{d}l \:\mathrm{d}t + C\_{D\Omega}\Omega= 0

\label{torque\_1}

\end{equation}

Where $\Delta T$ is the time filament motion repeats and integration is taken over the whole lenght $(L)$ of the

helix.

\paragraph{}

In order to solve the integration problem, the force ($f$) is required to be defined. Therefore, a new

coordination system was introduced and the force vector was defined as a composition of force per unit length

in the normal and tangent directions. The two identical motions are considered for the swimming microrobots

are; rotating and translating (assumed in the x-axis direction). Hence, the force balance and moment balance

equations are simplified as follows;

\begin{equation}

Nf\_xL + C\_DU = 0

\label{simple\_thrust}

\end{equation}

\begin{equation}

Nf\_yAL + C\_{D\Omega}\Omega= 0

\label{simple\_torque}

\end{equation}

Where $N$ and $A$ are number of filaments and helical amplitude of filaments respectively. Also $f\_x$ and

$f\_y$ shows the components of the force vector along x and y directions. In addition, $C\_D$ and $C\_{D\Omega}$

were computed by equations \ref{Coeff1} and \ref{Coeff2} where $R$ is radius of the helix and $\mu$ is fluid velocity.

\begin{equation}

C\_D = 6 \pi \mu R

\label{Coeff1}

\end{equation}

\begin{equation}

C\_{D\Omega}= 8 \pi \mu R^3

\label{Coeff2}

\end{equation}

The $f\_x$ and $f\_y$ are written as composite of forces in the normal and tangent directions;

\begin{equation}

f\_x = f\_t\cos \theta - f\_n\sin \theta

\label{normal}

\end{equation}

\begin{equation}

f\_y = f\_t\sin \theta + f\_n\cos \theta

\label{tangant}

\end{equation}

\begin{equation}

\tan \theta = \cfrac{\lambda}{2\pi A}

\label{tang}

\end{equation}

\begin{equation}

f\_t = -C\_t(U \cos \theta - \omega A \sin \theta)

\label{normal\_f}

\end{equation}

\begin{equation}

f\_n = - C\_n(-U \sin \theta - \omega A \cos \theta)

\label{tangant\_f}

\end{equation}

Where $C\_t$ and $C\_n$ called resistance coefficients ~\citep{edd2003biomimetic};

\begin{equation}

C\_t = \cfrac{2 \pi \mu}{\ln (\cfrac{2 \lambda}{b}) - \cfrac{1}{2}}

\label{Coeffient1}

\end{equation}

\begin{equation}

C\_n = \cfrac{4 \pi \mu}{\ln (\cfrac{2 \lambda}{b}) + \cfrac{1}{2}}

\label{Coeffient1}

\end{equation}

Microrobot\rq{}s swimming speed and rotation rate were determined by solving

the equations \ref{normal\_f} and \ref{tangant\_f}. Therefore, thrust ($F$), torque ($T$) and drag ($D$)

on flagellum can be predict by following equations~\citep{rodenborn2013propulsion};

\begin{equation}

F = (\Omega R)(C\_n - C\_t) \sin \theta \cos \theta \cfrac{L}{\cos \theta}

\label{thrust}

\end{equation}

\begin{equation}

T = (\Omega R^2)(C\_n \cos ^2 \theta + C\_t \sin ^2 \theta) \cfrac{L}{\cos \theta}

\label{torque}

\end{equation}

\begin{equation}

D = U (C\_n \sin ^2 \theta + C\_t \cos ^2 \theta) \cfrac{L}{\cos \theta}

\label{drag}

\end{equation}

Finally, the efficiency of the helical swimmers can be computed as follow;

\begin{equation}

efficiency = \cfrac{FU}{T \omega}

\label{efficiency}

\end{equation}

\subsubsection{Resistive force theory for six degrees of freedom}\label{RFT\_sixDegree}

The two degrees of freedom microrobot (one dimension model) with \ac\*{RFT} was exhibited a successful result~\citep{mahoney2011velocity}

for studying the helical microswimmers. However, complex motion of swimming microrobot could not be

explained in one dimension model. Therefore, the \ac\*{RFT} was needed to be implemented in three dimension

model which means defining a microrobot with six degrees of freedom. The microrobot\rq{}s is

used in this model has a helical tail with the sphere head attached to it as shown in \ref{RFT-6dof}.

\begin{figure}

\centering

%\begin{wrapfigure}{r}{0.5\textwidth}

% \begin{center}

\includegraphics[width=1.0\textwidth]{RFT-6dof}

\caption[Helical microrobot configuration with a magnetic spherical head]{Three dimension configration for the helical microrobot with a magnetic spherical head. The origin

of the helix coordinate is denoted with $\bm{O}\_h$ and $\bm{x}\_{h}$ is the central axis of the

helix~\citep{mahoney2011velocity}.}

\label{RFT-6dof}

%\end{center}

%\end{wrapfigure}

\end{figure}

The \ac\*{RFT} is applied with the assumption that the force and torque is applied on the helical tail and sphear

head are independent. Therefore, the force $f\_h$ and torque $\tau\_h$ of the helical tail are obtained by \ac\*{RFT}

and $f\_m$ and $\tau\_m$ are force and torque applied on the sphear head respectively.

The equation \ref{total\_force\_torque} is the summation of two forces and torques which is the total force and torque.

\begin{equation}

f = f\_h + f\_m \qquad \tau = \tau\_h + \tau\_m

\label{total\_force\_torque}

\end{equation}

According to the \ac\*{RFT} the force on the extremely minuscule segment of the helix is defined by

the segment velocity and drag forces acts on that segment. First \ac\*{RFT} takes the

velocity ($\bm{V}\_s$) that was applied on the small length

of helix and decompound it into two vectors, one parallel ($\bm{V}\_{\parallel}$) and one perpendicular ($\bm{V}\_{\perp}$)

to that segment. Also, the drag force is acting on the small length decomposed into two

vectors; parallel (${\xi}\_{\parallel}$) and perpendicular (${\xi}\_{\perp}$) to that segment.

Therefore, the force is applied on the small segment is formulated as follow;

\begin{equation}

d{\bm{f}\_{\perp}} = {\xi}\_{\perp}{\bm{V}\_{\perp}}ds

\label{relation-force\_drag}

\end{equation}

\begin{equation}

d{\bm{f}\_{\parallel}} = {\xi}\_{\parallel}{\bm{V}\_{\parallel}}ds

\label{relation-force\_drag}

\end{equation}

Where ${\xi}\_{\parallel}$ and ${\xi}\_{\perp}$ are drag coefficients and they have been approximated by a number

of scientists emprically. The fluidic force ($ \bm{f}\_h$) acting on the helix is computed by integrating over these

differential forces along the helix length. Because the integration is performed in three dimensions

we need to define two seperate coordinate frame, one for the given

differential segment (frame $s$) and one for the helix (frame $h$). The helix pitch angle ($\theta$)

and radius ($r\_h$) is used to define the geometry of the helix with the assumption that the central

axe of the helix is

parallel to the $\bm{x}\_h$. Figure \ref{RFT-6dof} presents the helix coordinate origin ($O\_h$) with

its three axis ($\bm{x}\_h , \bm{y}\_h , \bm{z}\_h$). The helix is represented in a cylindrical coordinate

\footnote{The cylindrical coordinate is an extention of the polar coordinate to the three dimension space. It

defines based on the radius $r$, the angle $\theta$ and the $z$ coordinate such that the following equation

are valid

\begin{equation}

x = r\cos{\theta} \qquad y = r\sin{\theta} \qquad z = z

\label{cylindrical\_coordinate}

\end{equation}

}

system with the polar angle $\phi$. Each vector in the segment frame ($s$) can be written in the helix

frame ($h$) by applying a rotation matrix as shown in the equation \ref{rotation\_matrix};

\begin{equation}

^{h}\bm{R}\_s(\phi) = \bm{R}\_x(\phi)\bm{R}\_y(-\theta)

\label{rotation\_matrix}

\end{equation}

Where $\bm{R}\_x(\phi)$ is rotation of a vector in the segment frame ($s$) with the $\phi$ angle with

respect to the $x$

axis and then will apply $\bm{R}\_y(-\theta)$ which rotate the result vector with the ($-\theta$) angle with

respect to the $y$ axis. The final result is a vector in the helix ($h$) frame \ref{rotated\_vector}.

\begin{equation}

^{h}\bm{P}(\phi) = \begin{bmatrix}

\frac{r}{\tan(\theta)} & r\cos(\phi) & r\sin(\phi) \\[0.3em]

\end{bmatrix}

\label{rotated\_vector}

\end{equation}

Hence, as appears in the equation \ref{velocity\_force\_segment\_frame} the differential relating velocity to force can be shown with respect to the frame of

a random segment along the helix in the segment frame in three dimension.

\begin{equation}

^{s}d\bm{f}\_s = {^{s}\bm{\Xi}}{^{s}\bm{V}\_s}{ds}

\label{velocity\_force\_segment\_frame}

\end{equation}

\begin{equation}

^{s}\bm\Xi = \begin{bmatrix}

\xi\_{\parallel} & 0 & 0 \\[0.3em]

0 & \xi\_{\perp} & 0\\[0.3em]

0 & 0 & \xi\_{\perp}

\end{bmatrix}

\label{drag\_coeffi\_matrix}

\end{equation}

In the equation \ref{velocity\_force\_segment\_frame} the force $^{s}\bm{f}\_s$ and velocity $^{s}\bm{V}\_s$ of

the segment is represented in the segment\rq{}s own frame.

In the segment frame, the $x\_s$ axis is assumed to be parallel to that segment and two other axis

($y\_s$, $z\_s$) are perpendicular to that segment as we can see in the \ref{drag\_coeffi\_matrix}.

Hence, the relationship between forces and velocity can be expressed in the helix

frame (\ref{velocity\_force\_helix\_frame}) by using the drag coefficient unity matrix \ref{drag\_coeffi\_matrix}.

\begin{equation}

^{h}d\bm{f}\_s = {^{h}\bm{\Xi}}(\phi){^{h}\bm{V}\_s}{ds}

\label{velocity\_force\_helix\_frame}

\end{equation}

where

\begin{equation}

{^{h}\bm{\Xi}}(\phi) = ^{h}\bm{R}\_s(\phi){^{s}\bm{\Xi}}{^{s}\bm{R}\_h(\phi)}

\label{drag\_coeff\_matrix\_rotated}

\end{equation}

The velocity of the small helix segment $\bm{V}\_s$ is formed of the rotational helix velocity ($\omega$)

and its translational velocity ($\bm{V}$). The summation of the two velocities is described in the equations

\ref{total\_velocity}.

\begin{equation}

\bm{V}\_s = \bm{V} + \bm{\omega} \times {\bm{P}(\phi)} = \bm{V} - {\bm{P}(\phi)}\times{\bm{\omega}}

\label{total\_velocity}

\end{equation}

The equation \ref{total\_velocity} is the velocity of the segment in the segment frame.

This equation can be written with respect of the helix frame, as shown below;

\begin{equation}

^{h}\bm{V}\_s = ^{h}\bm{V} - \Delta{\{^{h}\bm{P}(\phi)}\}^{h}\bm{\omega} = ^{h}\bm{V} + \Delta{\{^{h}\bm{P}(\phi)}\}^{Th}\bm{\omega}

\label{total\_velocity\_helixFrame}

\end{equation}

where the vector cross product (${\bm{P}(\phi)}\times{\bm{\omega}}$) can be represented in the form

of skew-symmetric

martix \footnote{In mathematics, a square matrix $A$ is called a skew-symmetric if its transpose

is equal to its negative ($A^{T} = -A$).} $\Delta{\{^{h}\bm{P}(\phi)}\}^{h}$ and a vector

$\bm{\omega}$:

\begin{equation}

{\bm{P}(\phi)}\times{\bm{\omega}} = \Delta{\{^{h}\bm{P}(\phi)}\}^{h}{\bm{\omega}}

\label{cross\_product}

\end{equation}

And acording to the skew-symmetric matrix property we have:

\begin{equation}

-\Delta{\{^{h}\bm{P}(\phi)}\}^{h} = \Delta{\{^{h}\bm{P}(\phi)}\}^{Th}

\label{skew\_symetric\_vector}

\end{equation}

After substituting \ref{total\_velocity\_helixFrame} into \ref{velocity\_force\_helix\_frame}:

\begin{equation}

^{h}d\bm{f}\_s = {^{h}\bm{\Xi}}(\phi){^{h}\bm{V}}{ds} + ^{h}\Xi(\phi)\Delta{\{^{h}\bm{P}(\phi)}\}^{Th}\omega{ds}

\label{Final\_force\_related\_RotationTranslation}

\end{equation}

The equation \ref{Final\_force\_related\_RotationTranslation} manifests the relationship between differential force

and translation and rotation velocity of the small helix segment in the helix frame. Each force is applied on

an infinitesimally small section of helix generates a torque around helix centre. As a result the

relation between the force and torque at an arbitary slice of helix (using parameter $\phi$) can be

represented in the helix frame:

\begin{equation}

^{h}d\bm{\tau}\_s = {^{h}\bm{P}(\phi)} \times ^{h}d\bm{f}\_s=\Delta{\{^{h}\bm{P}(\phi)}\}^{h}{d\bm{f}\_s}

\label{forceTorque\_relation\_helixFrame}

\end{equation}

Therefore the total fluidic torque and force of the helix can be figured out by integrating the small torques

and forces that applied to the extremely small segments of the helix along the helix length:

\begin{equation}

\bm{f}\_h = \int \; d\bm{f}\_s \qquad \bm{\tau}\_h = \int \; d\bm{\tau}\_s

\label{total\_force\_torque}

\end{equation}

The final torque and force can be obtained from the equations \ref{total\_force\_torque} by

integrating with respect to the polar angle $\phi$. As it been seen in the figure \ref{RFT-6dof} the $ds$

can be written with respect to the polar angle $\phi$ as follow:

\begin{equation}

ds = \frac{r\_hd\phi}{\sin(\theta)}

\label{polar\_angle\_theta}

\end{equation}

after substitiuting the \ref{Final\_force\_related\_RotationTranslation} into \ref{forceTorque\_relation\_helixFrame} and

replacing $ds$ with the eqation \ref{polar\_angle\_theta} we have the following equations which is

integrating with respect with $\phi$ from $-\pi n$ to $\pi n$ for an $n$ turn helix;

\begin{multline}

\qquad \qquad^{h}\bm{f}\_h = \left ( \frac{r\_h}{\sin(\theta)} \int \_{-\pi n}^{\pi n} \mathrm {^{h}\Xi(\phi)}\, d(\phi) \right){^{h}{\bm{V}}} \\

+ \left ( \frac{r\_h}{\sin(\theta)} \int \_{-\pi n}^{\pi n} \mathrm {^{h}\Xi(\phi)} \Delta{\{^{h}\bm{P}(\phi)}\}^T \, d(\phi) \right){^{h}{\bm{\omega}}}

\label{first\_intergra\_force}

\end{multline}

\begin{multline}

\qquad \qquad ^{h}\bm{\tau}\_h = \left ( \frac{r\_h}{\sin(\theta)} \int \_{-\pi n}^{\pi n} \mathrm \Delta{\{^{h}\bm{P}(\phi)}\} {^{h}\Xi(\phi)}\, d(\phi) \right){^{h}{\bm{V}}} \\

+ \left ( \frac{r\_h}{\sin(\theta)} \int \_{-\pi n}^{\pi n} \mathrm \Delta{\{^{h}\bm{P}(\phi)}\} {^{h}\Xi(\phi)} \Delta{\{^{h}\bm{P}(\phi)}\}^T \, d(\phi) \right){^{h}{\bm{\omega}}}

\label{second\_integral\_torque}

\end{multline}

Computing all four integrals in the equations \ref{first\_intergra\_force} and \ref{second\_integral\_torque} will

result in two eqations that is expressed force $(^{h}\bm{f}\_h)$ and torque $(^{h}\bm{\tau}\_h)$ in

terms of the angular $(^{h}\bm{\omega})$ and translational velocity $(^{h}\bm{V})$:

\[

\begin{bmatrix} ^{h}\bm{f}\_h\\

^{h}\bm{\tau}\_h\end{bmatrix} = \begin{bmatrix} ^{h}\bm{A}\_h & ^{h}\bm{B}\_h \\

^{h}\bm{C}\_h & ^{h}\bm{D}\_h \end{bmatrix} \begin{bmatrix} ^{h}\bm{V}\_h

\\ ^{h}\bm{\omega}

\end{bmatrix}

\]

Where $^{h}\bm{A}\_h$, $^{h}\bm{B}\_h$ and $^{h}\bm{C}\_h$ are:

\begin{equation}

^{h}\bm{A}\_h = \begin{bmatrix}

a\_{h11} & 0 & 0 \\[0.3em]

0 & a\_{h22} & 0\\[0.3em]

0 & 0 & a\_{h22}

\end{bmatrix}

\label{Amatrix}

\end{equation}

\begin{equation}

^{h}\bm{B}\_h = \begin{bmatrix}

b\_{h11} & 0 & b\_{h13} \\[0.3em]

0 & b\_{h22} & 0\\[0.3em]

0 & 0 & b\_{h33}

\end{bmatrix}

\label{Bmatrix}

\end{equation}

\begin{equation}

^{h}\bm{C}\_h = \begin{bmatrix}

c\_{h11} & 0 & c\_{h13} \\[0.3em]

0 & c\_{h22} & 0\\[0.3em]

c\_{h13} & 0 & c\_{h33}

\end{bmatrix}

\label{Cmatrix}

\end{equation}

and each matrix element will calculate by following eqations:

\begin{equation}

a\_{h11} = \frac{2\pi n r\_h (\xi\_{\parallel} \cos^2(\theta) + \xi\_{\perp} \sin^2(\theta))}{\sin(\theta) }

\label{ah11}

\end{equation}

\begin{equation}

a\_{h11} = \frac{\pi n r\_h (\xi\_{\perp} + \xi\_{\perp} \cos^2(\theta) + \xi\_{\parallel} \sin^2(\theta))}{\sin(\theta) }

\label{ah22}

\end{equation}

\begin{equation}

b\_{h11} = 2\pi n {r\_h}^2 (\xi\_{\parallel} - \xi\_{\perp})\cos(\theta)

\label{bh11}

\end{equation}

\begin{equation}

a\_{h13} = \frac{-2 \pi n {r\_h}^2 (\xi\_{\parallel} - \xi\_{\perp})\cos(\theta)}{\tan(\theta) }

\label{bh13}

\end{equation}

\begin{equation}

a\_{h22} = \frac{-3 \pi n {r\_h}^2 (\xi\_{\parallel} - \xi\_{\perp})\cos(\theta)}{2}

\label{bh22}

\end{equation}

\begin{equation}

a\_{h33} = \frac{- \pi n {r\_h}^2 (\xi\_{\parallel} - \xi\_{\perp})\cos(\theta)}{2}

\label{bh33}

\end{equation}

\begin{equation}

c\_{h11} = \frac{2\pi n {r\_h}^3 (\xi\_{\perp} \cos^2(\theta) + \xi\_{\parallel} \sin^2(\theta))}{\sin(\theta) }

\label{ch11}

\end{equation}

\begin{equation}

c\_{h11} = \frac{-2\pi n {r\_h}^3 (\xi\_{\perp} \cos^2(\theta) + \xi\_{\parallel} \sin^2(\theta))}{\sin(\theta) \tan(\theta)}

\label{ch13}

\end{equation}

\begin{multline}

c\_{h22} = \frac{2\pi n {r\_h}^3 (\xi\_{\parallel} \cos^2(\theta) + \xi\_{\perp} \sin^2(\theta) - \xi\_{\perp}/2)}{\sin(\theta) } \\

+ \frac{\pi n {r\_h}^3 (\xi\_{\parallel} \cos^2(\theta) - \xi\_{\perp} \sin^2(\theta) - \xi\_{\perp})}{2{\tan^2(\theta)}\sin(\theta)}\\

+ \frac{(\pi n {r\_h})^3 (\xi\_{\parallel} \cos^2(\theta) - \xi\_{\perp} \sin^2(\theta) + \xi\_{\perp})}{3{\tan^2(\theta)}\sin(\theta)}

\label{ch22}

\end{multline}

\begin{multline}

c\_{h33} = \frac{\pi n {r\_h}^3 \xi\_{\perp} }{\sin(\theta) }- \frac{\pi n {r\_h}^3 (\xi\_{\perp} \cos^2(\theta) + \xi\_{\parallel} \sin^2(\theta) - \xi\_{\perp})}{2{\tan^2(\theta)}\sin(\theta)}\\

+ \frac{(\pi n {r\_h})^3 (\xi\_{\perp} \cos^2(\theta) + \xi\_{\parallel} \sin^2(\theta) + \xi\_{\perp})}{3{\tan^2(\theta)}\sin(\theta)}

\label{ch33}

\end{multline}

We assumed the fluidic torque and force are applied on microrobot by helical tail is independent from the

spherical head. We define a vector $\bm K$ such that it connects the centre of the helix $\bm O\_h$ to the

centre of the spherical magnetic head $\bm O\_m$ as shown in the Figure~\ref{RFT-6dof}. The well-known

equations for the rotational and translational drag coefficient of the sphear particle in the stokes flow

are \citep{white1991viscous}:

\begin{equation}

\xi\_{vm} = 6 \pi \eta r\_m \qquad \qquad \xi\_{\omega m} = 8 \pi \eta r^3\_m

\label{shearical\_drag\_coefficients}

\end{equation}

Where $\eta$ is the fluid viscosity and $r$ is the radius of the sphear. A magnet velocity is produced by an

arbitrary movement of the microswimmer and can be expressed in the helix frame as the product of the

head\rq{}s velocity and translational drag coefficient:

\begin{equation}

^{h}\bm V\_{m} = ^{h}\bm{V} + ^{h}\bm{\omega} \times ^{h}\bm{ K }= ^{h}\bm {V} - ^{h}\bm{K} \times ^{h}\bm{\omega}

= ^{h}\bm{V} + \Delta \{^{h}\bm{ K} \}^{Th} \bm{\omega}

\label{magnet\_velocity}

\end{equation}

Also, force on the spherical magnet is the product of the translational and rotational force:

\begin{equation}

^{h}\bm{f}\_{m} = \xi\_{vm} {^{h}{\bm{V}}} + \xi\_{vm} \Delta\{ {^{h}\bm{K}}\}^{Th} \bm{\omega}

\label{magnet\_force}

\end{equation}

The force acts at the arm $\bm{K}$ and the drag is generated by the rotation of the spherical magnet will

couse a drag torque by magnet head:

\begin{equation}

^{h}\bm{\tau}\_{m} = ^{h}\bm{K} \times ^{h}\bm{f}\_{m} + \xi\_{\omega m} {^{h}\bm{\omega}}

\label{magnet\_head\_torque}

\end{equation}

After replacing $^{h}\bm{f}\_{m}$ with \ref{magnet\_force} and using scew-symmetric matrix instead of

cross-product, the final torque for magnetic head will be:

\begin{equation}

^{h}\bm{\tau}\_{m} = \xi\_{vm}\Delta\{ {^{h}\bm{K}}\} {^{h}{\bm{V}}} + (\xi\_{vm} \Delta\{ {^{h}\bm{K}}\} {\Delta\{ {^{h}\bm{K}}\}}^{T} + \xi\_{\omega m} \bm{I} ){^{h}\bm{\omega}}

\label{magnet\_torque\_final}

\end{equation}

We can write the equation \ref{magnet\_torque\_fina} in terms of matrices;

\begin{equation}

^{h}\bm{A}\_m = \xi\_{vm} \bm{I} \qquad ^{h}\bm{B}\_m = \xi\_{vm} \Delta \{ ^{h}\bm{K} \}^T \qquad

{^{h}\bm{B}\_m = \xi\_{vm} \Delta \{ ^{h}\bm{K} \} \Delta \{ ^{h}\bm{K} \}^T} + \xi\_{\omega m} \bm{ I}

\label{A\_m}

\end{equation}

Therefore, the total torque ($^{h}\bm{\tau} =^{h}\bm{\tau}\_h +^{h}\bm{\tau}\_m$) and force

($^{h}\bm{f} =^{h}\bm{f}\_h +^{h}\bm{f}\_m$) applied on microswimmer are:

\[

\begin{bmatrix} ^{h}\bm{f}\\

^{h}\bm{\tau}\end{bmatrix} = \begin{bmatrix} ^{h}\bm{A} & ^{h}\bm{B}\\

^{h}\bm{B}^{T} & ^{h}\bm{C} \end{bmatrix} \begin{bmatrix} ^{h}\bm{V}

\\ ^{h}\bm{\omega}

\end{bmatrix}

\]

By replacing the matrices with their equivalent;

%\begin{equation}

\[

\begin{bmatrix} ^{h}\bm{f}\\

^{h}\bm{\tau}\end{bmatrix} = \begin{bmatrix} ^{h}\bm{A}\_{h} + ^{h}\bm{A}\_{m} & {^{h}\bm{B}\_{h} + ^{h}\bm{B}\_{m} }\\

({^{h}\bm{B}\_{h} + ^{h}\bm{B}\_{m} })^{T} & ^{h}\bm{C}\_{h} + ^{h}\bm{C}\_{m} \end{bmatrix} \begin{bmatrix} ^{h}\bm{V}

\\ ^{h}\bm{\omega}

\end{bmatrix}

\]

%\label{FINAL\_PROPULSION}

%\end{equation}

\begin{equation}

^{h}\bm{A} = \begin{bmatrix}

a\_{11} & 0 & 0 \\[0.3em]

0 & a\_{22} & 0\\[0.3em]

0 & 0 & a\_{22}

\end{bmatrix}

=

\begin{bmatrix}

a\_{h11}+\xi\_{vm} & 0 & 0 \\[0.3em]

0 & a\_{h22}+\xi\_{vm} & 0\\[0.3em]

0 & 0 & a\_{h22}+\xi\_{vm}

\end{bmatrix}

\label{A\_finalmatrix}

\end{equation}

\begin{equation}

^{h}\bm{B} = \begin{bmatrix}

b\_{11} & 0 & b\_{13} \\[0.3em]

0 & b\_{22} & b\_{23}\\[0.3em]

0 & -b\_{23} & b\_{33}

\end{bmatrix}

=

\begin{bmatrix}

b\_{h11} & 0 & b\_{h13} \\[0.3em]

0 & b\_{h22} & \xi\_{vm}|\bm{K}| \\[0.3em]

0 & - \xi\_{vm}|\bm{K}| & b\_{h33}

\end{bmatrix}

\label{B\_finalmatrix}

\end{equation}

\begin{equation}

^{h}\bm{C} = \begin{bmatrix}

c\_{11} & 0 & c\_{h13} \\[0.3em]

0 & c\_{22} & 0\\[0.3em]

c\_{h13} & 0 & c\_{33}

\end{bmatrix}

=

\begin{bmatrix}

c\_{h11}+ \xi\_{\omega m} & 0 & c\_{h13} \\[0.3em]

0 & c\_{h22}+ \xi\_{vm}|\bm{K}|^2 +\xi\_{\omega m} & 0\\[0.3em]

c\_{h13} & 0 & c\_{h33} + \xi\_{vm}|\bm{K}|^2 +\xi\_{\omega m}

\end{bmatrix}

\label{C\_Finalmatrix}

\end{equation}

Hence, the total nonfluidic force ($\bm{f}$) which produced as a result of gravity and total nonfluidic torque

($\bm{\tau}$) generated by magnetic field.

\subsubsection{Regularized Stokeslet method}\label{method2}

A regularization parameter can be used as a proxy for the body radius to minimise numerical errors

in modelling a flagellum as a one dimensional filament in a low Reynolds~\citep{smith2009boundary}.

number fluids. The Regularized Stokeslet method (RSM) is one of the approch to solve the zero

Reynolds number linear \lq{}Stokes flow\rq{} equations;

\begin{equation}

\left.\begin{aligned}

0 &= - \nabla p + \mu \nabla ^ 2 u + f \\

0 &=\nabla . u

\end{aligned}

\right\}

\qquad \text{Stokes flow equations}

\label{stokes}

\end{equation}

where $u$ (velocity), $p$ (pressure), $\mu $ (kinematic viscosity), and $f$ (force) are measured per unit

volume. The singular \lq{}Stokeslet\rq{} solution for the equations \ref{stokes} corresponds to the purely

viscous component (point force) of the flow, which was determined by moving sphere.

The \lq{}Stokeslet\rq{} solution for unit force acts in the j-direction and concentrated at $\xi $, where $f$ is;

\begin{equation}

f (x) = \delta (x - \xi) e\_j

\label{force}

\end{equation}

$\delta (x - \xi)$ is called Dirac delta distribution. The velocity in the i-direction driven by this force is defined

as follow;

\begin{equation}

S \_{ij} \bm{(x , \xi)} = (\frac{\delta \_{ij}}{r} + \frac{r\_i r\_j}{r^3})

\label{i-direction}

\end{equation}

Where $\delta \_{ij}$ denotes Kronecker delta tensor, $r\_i = x\_i - \xi \_i$ and $r^2 =| \bm{x}- \bm{\xi} |^2 = r\_1 ^2 + r\_2 ^ 2 + r\_3 ^2$.

The flow concentrates at point $\bm \xi$ by the force $\bold F$ where $ \bm{f (x)} = \delta (\bm{x} - \bm{\xi) F}$.

The solution is given by finding the velocity $u\_i(\bm x)$;

\begin{equation}

u\_i(x) =(\frac{1}{8 \pi \mu}) S\_{ij}\bm{( x, \xi)} F\_j

\label{velocity}

\end{equation}

The \ac\*{RFT} and \ac\*{SBT} for modelling of flagellum driven

flow were formed on the base of the Stokeslet~\citep{smith2009boundary}. These methods solved the

three dimentinal flow problem with flexible boundaries without using direct computation for the differential

equations. Therefore, the provided solutions are extremely efficient in terms of computational costs. The fluid

velocity was modeled by the following equation;

\begin{equation}

\bold {u(x)} =(\frac{1}{8 \pi \mu}) \int\_S \mathrm \bm{f(\xi)} . \bm{S( x, \xi)}\, \mathrm{d}S\_{\xi}

\label{fluid\_velocity}

\end{equation}

Where $S$ is a collection of lines or surfaces of flagella, $\bm{f(\xi)}$ shows force per unit length or area.

$\bm{f(\xi)} dS\_{\xi}$ denotes the force flagella body exerted on the fluid and $-\bm{f(\xi)} dS\_{\xi}$ is the

force fluid applies to the body. The flagella is represented by equation \ref{fluid\_velocity} with the boundary

$S$ and parameter $\bm{\xi (s)}$ where $0 < s <1$ is scaled arclength parameter. However, the flow field

at any point $\bm{x} = \bm{\xi} (s)$ is sigular and the collection of points on the surface of the filament

are required to calculate the force per unit length. The collection of points were replaced on a small distance

from the centreline;

\begin{equation}

\bold {X(s\_q)} = \bm{\xi}(s\_q) + a(s\_q) \bm{n}(s\_q)

\label{centreline}

\end{equation}

where $a(s\_q)$ is a radius of slender body and $\bm{n}(s\_q)$ is a unit normal vector. Point distributions of

Stokeslets at any point $x = \bm{\xi}\_q$ and line distribution inside the notional surface

of the flagella are both singular. However, surface distributions of Stokeslets do not result in singular velocity

but still requires attentive numerical implementations \citep{smith2009boundary}. The \lq{regularized Stokeslet}\rq{}

introduced an exact solution for the equations \ref{$stokes\_reg$} to overcome these issues.

This method used a cut off function ($\psi$) with a regularization parameter ($\epsilon$) to smooth point forces

such that $\int\_{R^3} \psi\_{\epsilon} (\bm{x})\, \mathrm{d}V\_x = 1$.

\begin{equation}

\left.\begin{aligned}

0 &= - \bm{\nabla} p + \mu \nabla ^ 2 \bm{u} + \bm{f} \psi\_\epsilon (\bm{x} - \bm{\xi}), \\

0 &=\bm{\nabla . u}

\end{aligned}

\right\}

\qquad \text{Stokes flow equations with regularization parameter}

\label{stokes\_reg}

\end{equation}

In RSM method, with a assumption of $\psi\_{\epsilon}(\bm{x} - \bm{\xi}) := 15\epsilon^4 /8\pi \mu r\_{\epsilon}^7$

and $r\_{\epsilon} = \sqrt{r^2 + \epsilon ^2}$ the regularized Stokeslet velocity tensor measured by the the following:

\begin{equation}

S \_{ij}^{\epsilon} \bm{(x , \xi)} = \frac{\delta \_{ij}(r^2 + 2{\epsilon}^2) + r\_i r\_j}{r\_{\epsilon}^3}

\label{velocity-tensor}

\end{equation}

Therefore the boundary for intergal equation (\ref{fluid\_velocityReg}) is defined and the fluid velocity at location $x$ is;

\begin{equation}

\bold {u(x)} =(\frac{1}{8 \pi \mu}) \int\_S \mathrm \bm{f(\xi)} . \bm{S^{\epsilon}( x, \xi)}\, \mathrm{d}S\_{\xi}

\label{fluid\_velocityReg}

\end{equation}

Where $\bm{f(\xi)}$ denotes a fluidic force per unit area or length depends on $\bm{\xi}$. $\bm{\xi}$ could be on a

line or on a surface, in both cases for $\bm{x} = \bm{\xi}$ kernel is regular. This is a significant advantage of

\ac\*{RSM} to model swimming motion of microhelix in a high viscose fluid environment.

\begin{figure}

\centering

%\begin{wrapfigure}{r}{0.5\textwidth}

% \begin{center}

\includegraphics[width=0.80\textwidth]{Stoks}

\caption[RSM and SBT]{\ac\*{RSM} and \ac\*{SBT}. In \ac\*{RSM} the surface of helix filament is separated by

cross-sectional segmentation and each surface represents by

Stokeslets (left image). In \ac\*{SBT}, the Stokeslets are arranged along the

central filament line (rigth image)~\citep{rodenborn2013propulsion}.}

\label{Stoks}

%\end{center}

%\end{wrapfigure}

\end{figure}

\subsubsection{Slender body theory}\label{method1}

Slender body theory represents the helix body with an arrangement of

doublets and Stokeslets along the filament central line (Figure \ref{Stoks}). This theory represented by

Lighthill for the first time and was followed by Johnson with some modification. According to \citeauthor{lighthill1971large}

there is some distance $q$ from any given point on the helix body such that $q$ is between the radius of the

filament $a$ and helix pitch $\lambda$. The dipoles is fallen within this distance are important in determining the flow at the

given point. He proved for the induced fluid flow on the given segment

the sum of near-field and far-field solutions could be made independent of any distance ($q$) by definding

the dipoles as follow;

\begin{equation}

-\frac{a^2 \bm{f}\_\perp (s)}{4\mu}

\label{dipole}

\end{equation}

Where $s$ is a location along the central axis of the filament and $\bm{f}\_\perp (s)$ is Stokeslets strength\rq{}s

component which is perpendicular to the filament central axis. Therefore the sum of dipole and Stokeslets

define the flow induced by each segment of the helix body. As a result there is a relation between

the local velocity of a segment on the $s$ location and the force per unit length;

\begin{equation}

\bm{u}(s) = -\frac{a^2 \bm{f}\_\perp (s)}{4\mu} + \int\_{|\bm{r\_0}(s\rq{},s)| > \delta } \bm{f}(s\rq{}).J(\bm{r\_0})\: \mathrm{d}s\rq{}

\label{dipole\_stokes}

\end{equation}

Where $\bm{r\_0}$ is the vector from the point $s$ on the central axis to the point $s\rq{}$ and $\delta$ is

a natural cutoff ($\delta = \frac{a\sqrt{e}}{2}$). For the spatial location $r$ Oseen tensor $J$ is;

\begin{equation}

J(\bm{r}) \equiv \frac{1}{8\pi \mu} (\frac{I}{|\bm{r}|} + \frac{\bm{rr}^T}{|\bm{r}|^3})

\label{Oseen}

\end{equation}

The thrust, torque and drag of the helical microswimmer can be optained by applying rectangular rule of

numerical integration and as a result we have;

\begin{equation}

J(\bm{r}) \equiv \frac{1}{8\pi \mu} (\frac{I}{|\bm{r}|} + \frac{\bm{rr}^T}{|\bm{r}|^3})

\label{Oseen}

\end{equation}

We need to parameterize spatial locations, so we define helical phase $\phi \equiv ks \cos(\theta)$ where

$k = 2 \pi / \lambda$ and $\bm{r} = R(\phi \cot(\theta), \cos(\phi) , \sin(\phi))$. Therefore, equation \ref{dipole\_stokes}

is converted to following equation;

\begin{equation}

\bm{u}\_n = \frac{(I- \hat{t}\_n\hat{t}\_n + D\_n). \bm{f}\_n}{4\pi \mu}

+ \frac{\bm{R}\bm{\Delta}\phi \csc (\theta)}{8 \pi \mu} \sum\_{m \neq n} \frac{I+\hat{r}\_{nm} \hat{r}\_{nm}}{r\_{nm}}. \bm{f}\_m

+ \Lambda (\Delta \phi)

\label{numerical}

\end{equation}

Where $m,n = 1,2, \dots ,N$ and $\hat{t}\_n=(\cos(\theta), -\sin(\theta)\sin(\phi\_n), \sin(\theta)\cos(\phi\_n))$.

The position vector between spatial location is $\bm{r}\_{nm} = \bm{r}(\phi\_n)- \bm{r}(\phi\_m)$.

The components of the velocity $\bm{u}\_n$ that are invariant alongside the helix can be obtained by

integrating over the eqation \ref{numerical} and using the frame rotated with the helical phase. Then we can

find the linear mapping between force and velocity per unit length and calculate the rotational and translation velocity

to find the force, torque and drag.

The first part of the equation \ref{numerical} is called tensor $D\_n$ and it shows the helical segments that

are centered at $\bm{r}$. $D\_n$ can be expressed in the form of following integral;

\begin{equation}

D\_n = 1/2 \int\_{|\bm{r}-\bm{r\_n}| \in (\delta, \delta \rq{})} ds(\phi) \big(\frac{I}{|\bm{r}-\bm{r}\_n|} + \frac{(\bm{r} -\bm{r}\_m)(\bm{r} -\bm{r}\_m)}{|\bm{r}-\bm{r}\_n|^3} \big). \chi\_z(\phi-\phi\_n)

\label{functionD}

\end{equation}

Where the rotation matrix $\chi\_z$ is defined as;

\begin{equation}

\chi\_z = \begin{bmatrix}

\cos(\phi) & -\sin(\phi) & 0 \\[0.4em]

\sin(\phi) & \cos(\phi) & 0\\[0.4em]

0 & 0 & 1

\end{bmatrix}

\label{rotationOperator}

\end{equation}

We define new vectors for force and velocity to simplify the calculation;

\begin{equation}

{\bm{u}\_n}\rq{}= \chi\_z(-\phi\_n). \bm{u}

\label{NewVelo}

\end{equation}

\begin{equation}

{\bm{f}\_n}\rq{}= \chi\_z(-\phi\_n). \bm{f}

\label{NewForce}

\end{equation}

Therefore, the velocity ${\bm{u}\_n}\rq{}$ is invariant to the filament and the rotational and translational velocity

of the helix can be written as;

\begin{equation}

{\bm{u}\_n}\rq{}= (0, \bm{\Omega} \bm{R} , \bm{U})^T

\label{NewVelo1}

\end{equation}

And the force;

\begin{equation}

\sum\_{i=1} {\bm{f}}\rq{} \bm{R} \bm{\Delta}\phi \csc \theta = {\big(0, \bm{T}/\bm{R}, \bm{F}\_x \big)}^T

\label{Newforce1}

\end{equation}

Therefore, \ac\*{SBT} can be expressed as;

\begin{multline}

\qquad {\bm{u}\_n}\rq{}= \frac{(I- \hat{t}\_n\hat{t}\_n + D\_n). \bm{f}\_n}{4\pi \mu}\\

+ \frac{\bm{R}\bm{\Delta}\phi \csc (\theta)}{8 \pi \mu} \sum\_{m \neq n} \frac{ \chi\_z(\phi\_m - \phi\_n)+ \chi\_z(-\phi\_n).\hat{r}\_{nm}\hat{r}\_{nm}. \chi\_z(-\phi\_n)}{r\_{nm}}. {\bm{f}\_m}\rq{}\\

+ \Lambda (\Delta \phi) \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad

\label{NewVelo2}

\end{multline}

Where both ${\hat{t}}\rq{}$ and ${D\_n}\rq{}$ are invariant to the helical filament,

\begin{equation}

{\hat{t}}\rq{}= (0, \sin \theta, \cos \theta)

\label{invariant}

\end{equation}

and

\begin{equation}

\int\_{k \delta \cos \theta}^{k {\delta}\rq{} \cos \theta} \mathrm d\phi \frac{1}{\phi}(I +\begin{pmatrix}

0 & 0 & 0 \\

0 & \sin^2\theta & \sin\theta\cos\theta \\

0 & \sin\theta\cos\theta & \cos^2\theta

\end{pmatrix})=\ln(\frac{\delta\rq{}}{\delta})(I + {\hat{t}}\rq{} {\hat{t}}\rq{})

\label{FinalD}

\end{equation}

So we obtained the mapping between the force and velocity;

\begin{equation}

\begin{pmatrix}

{\bm{u}\_1}\rq{} \\

{\bm{u}\_2}\rq{} \\

\vdots \\

{\bm{u}\_N}\rq{}

\end{pmatrix} = \Delta . \begin{pmatrix}

{\bm{f}\_1}\rq{} \\

{\bm{f}\_2}\rq{} \\

\vdots \\

{\bm{f}\_N}\rq{}

\end{pmatrix}

\label{mapping}

\end{equation}

For the velocity ${\bm{u}\_n}\rq{} = {\bm{u}\_0} = (0, \bm{\Omega} \bm{R}, \bm{U})^T$ we have;

\begin{equation}

\begin{pmatrix}

{\bm{f}\_1}\rq{} \\

{\bm{f}\_2}\rq{} \\

\vdots \\

{\bm{f}\_N}\rq{}

\end{pmatrix} = {\Delta}^{-1}. \begin{pmatrix}

{\bm{u}\_0} \\

{\bm{u}\_0} \\

\vdots \\

{\bm{u}\_0}

\end{pmatrix}

\label{mappingInverse}

\end{equation}

Finally, the fluidic force and torque are;

\begin{equation}

(0, \frac{T}{R}, F\_x)^T = \sum\_{i=1}^{N} \bm{f}\rq{} \bm{R} \bm{\Delta}\phi \csc \theta

\label{torqForce}

\end{equation}

\subsection{Microrobot actuation}\label{microActuation}

%\subsubsection{Force driven microrobot}

%\subsubsection{Torque driven microrobot}

In this section the aim is to develop an algorithm for the microrobot velocity control. To achieve this aim, we need to

fiqure out the direction that microrobot points out $(\bm{X}\_{h})$ and then its rotational speed $(\Omega)$

to obtain a desired velocity \cite{mahoney2011velocity}.

In this algorithm, the only nonfluidic force is applied on the microrobot is its weight which is expressed as

$m\bm{g}$. The mass of the microrobot is $m$ and the vector $\bm{g}$ shows the acceleration gravity.

The direction of the gravity is downward and represented by $\hat{ \bm{g} }= \bm{g}/ \| \bm{g}\|$.

%%%%%%%%%%%%%%% Flowchart of the control algorithm %%%%%%%%%%%%

Flowchart for Algorithm of the actuation method

%%%%%%%%%%%%

Previous research on controlling microswimmers\rq{}s speed is evident that there is a lack

of control if commanding microrobot with too rapid maneuvers \citep{zhang2009characterizing} \citep{zhang2009artificial}

. Therefore, in this work we assumed

microswimmers can turn continuously to the aimed direction in such a way that the temporary behaviour is

ignored. We define $\tilde{\bm{X}}$ as the axis magnetic field should always be perpendicular to it.

If the microrobot coordinate frame is aligned with the stationary world frame then there it does not need

to convert vectors between these two frames. From the helical propulsion equation system,

we specifically considered the first equation which is the relationship between non-fluidic force, angular and

translational velocity of the microswimmer.

\begin{equation}

^{h}\bm{f} = ^{h}\bm{A} ^{h}\bm{V} + ^{h}\bm{B} ^{h}\bm{\omega}

\label{first\_lineOf\_ propulsion method}

\end{equation}

The matrix $^{h}\bm{A}$ is invertible, thus the desired velocity can be obtained from the

equation \ref{first\_lineOf\_ propulsion method};

\begin{equation}

^{h}\bm{V} = (^{h}\bm{A} ^{-1}){^{h}\bm{f}} + (-^{h}\bm{A} ^{-1} {^{h}\bm{B}}){^{h}\bm{\omega}} =

^{h}\bm{D} ^{h}\bm{f} + ^{h}\bm{E} ^{h}\bm{\omega}

\label{first\_lineOf\_ propulsion method2}

\end{equation}

\begin{equation}

^{h}\bm{D}\_h = \begin{bmatrix}

d\_{11} & 0 & 0 \\[0.3em]

0 & d\_{22} & 0\\[0.3em]

0 & 0 & d\_{22}

\end{bmatrix}

\label{Dmatrix}

\end{equation}

\begin{equation}

^{h}\bm{E}\_h = \begin{bmatrix}

e\_{11} & 0 & 0 \\[0.3em]

0 & e\_{22} & 0\\[0.3em]

0 & 0 & e\_{22}

\end{bmatrix}

\label{Ematrix}

\end{equation}

The equations \ref{first\_lineOf\_ propulsion method2}, \ref{Dmatrix}, \ref{Ematrix} are in the helix frame and

can be converted to the world frame by applying relation matrix $^{w}\bm{R}\_h$ on the equation

\ref{first\_lineOf\_ propulsion method2};

\begin{equation}

{^{w}\bm{R}\_h}{^{h}\bm{V} } = {^{w}\bm{R}\_h}{ ^{h}\bm{D} ^{h}\bm{f}} + {^{w}\bm{R}\_h}{^{h}\bm{E} ^{h}\bm{\omega}}

\label{first\_lineOf\_ propulsion method3}

\end{equation}

\begin{equation}

^{w}\bm{V} ={^{w}\bm{E}} {^{w}\bm{\omega}} + {^{w}\bm{D}} {^{w}\bm{f}}

\label{first\_lineOf\_ propulsion method4}

\end{equation}

Then by applying the similar transformation to other component \ref{Dmatrix}, \ref{Ematrix};

\begin{equation}

^{w}\bm{V} = {^{w}\bm{R}\_h}{^{h}\bm{V}} \qquad ^{w}\bm{f} = {^{w}\bm{R}\_h}{^{h}\bm{f}}

\qquad ^{w}\bm{D} = {^{w}\bm{R}\_h}{^{h}\bm{D}} {^{h}\bm{R}\_w}

\qquad ^{w}\bm{E} = {^{w}\bm{R}\_h}{^{h}\bm{E}} {^{h}\bm{R}\_w}

\label{first\_lineOf\_ propulsion method5}

\end{equation}

To obtain ${^{h}\bm{R}\_w}$ the orientation of the microrobot needs to be detected

whilst it is rotating during propulsion around the axis which is difficult. For that reason, the equation

\ref{first\_lineOf\_ propulsion method4} is expressed in such way that does not need to know the

microrobot orientation whilst rotating about its central axis. Since the microrobot is torque driven and the only

nonfoluidic force is involved in equation \ref{first\_lineOf\_ propulsion method} is its weight $(m \bm{g})$. The

velocity of microrobot can be decomposed to vertical and horizontal components:

\begin{equation}

\bm{V}\_{ver} = (\bm{V . \hat{g}})\bm{\hat {g}}

\label{vertical\_velo}

\end{equation}

\begin{equation}

\bm{V}\_{hor} = \bm{V} - \bm{V}\_{ver}

\label{horiantal\_velo}

\end{equation}

Two options can be considered for the ${\| \bm {{V}\_{hor}}\|}$:

\begin{equation}

{\| \bm {{V}\_{hor}}\|} = 0 \qquad , \qquad {\| \bm{{V}\_{hor}}\|} \neq 0

\label{total\_force\_torque}

\end{equation}

The first option is a trival case, because when the microrobot is being commanded with

$ {\| \bm {{V}\_{hor}}\|} = 0$, that means the microrobot can only swim vertically in either

direction according to the equasion \ref{horiantal\_velo}. This is the special case when the six degrees of

freedom microrobot will effevtively become the microrobot with two degrees of freedom which is pointing

in the direction of the gravity acceleration and its angular velocity can be found directly from the eqation

\ref{first\_lineOf\_ propulsion method4}:

\begin{equation}

\bm {\Omega} = \frac{{\| \bm{V}\|}+ d\_{11}\| \bm{f}\|}{e\_{11}} \qquad , \qquad \tilde{\bm{X}} = -\hat{\bm{g}}

\label{angular\_velo\_horiVelo=0}

\end{equation}

The second option ${\| \bm{{V}\_{hor}}\|} \neq 0$ is more challenging, because it requires setting

the coordinate frame for microrobot which does not rotate when it is rotating around the central axis.

The ideal coordinate frame can be constructed by using $\hat{\bm{g}}$ and based on the eigenvectors

\footnote{If we have a set of data point, the set can be deconstructed into eigenvector and eigenvalue

where eigenvector is the direction that data spread out and eigenvalue is the variance of the data in that

direction. The principle component is the eigen vector with the largest eigenvalue \citep{Doe:2013Oct:Online}.} of

$^{w}\bm{D}$ or $^{w}\bm{E}$. This coordinate system is denoted by $p$ and can be defined as :

\begin{equation}

\bm{x}\_p = \frac{(\bm{{x}\_h . V )x\_h}}{|\bm{x\_h .V}|}

\label{x\_pAxis}

\end{equation}

\begin{equation}

\bm{y}\_p = \frac{(\bm{{x}\_p \times g)}}{\| \bm{x\_p \times g}\|}

\label{y\_pAxis}

\end{equation}

\begin{equation}

\bm{z}\_p = \bm{{x}\_p \times {y}\_p}

\label{z\_pAxis}

\end{equation}

\begin{figure}

\centering

%\begin{wrapfigure}{r}{0.5\textwidth}

% \begin{center}

\includegraphics[width=1.0\textwidth]{horiz\_verti\_velocity}

\caption[Construction details of

direction of the microswimmer]{(a) The principle coordinate frame based on the gravity and principle componets

of the matrices in equations \ref{Ematrix} and \ref{Dmatrix}. (b) Construction details of

direction of the microswimmer ($\bm{\tilde{X}}$)~\citep{mahoney2011velocity}.}

\label{horiz\_verti\_velocity}

%\end{center}

%\end{wrapfigure}

\end{figure}

The new (principle) coordinate system will solve the problem because it is invariant to the rotation

of the microswimer around its central axis. Therefore, the equation \ref{first\_lineOf\_ propulsion method4} can

be expressed in terms of the principle coordinate frame. In the following paragraph,

we first configure the representation for the first component (${^{w}\bm{E}} {^{w}\bm{\omega}}$) of the

equation \ref{first\_lineOf\_ propulsion method4}

and followed by a similar process on the second componet (${^{w}\bm{D}} {^{w}\bm{f}}$). The final result will express the

desired velocity vector in terms of the principal coodinate system.

It is assumed that the microrobot is at steady state, that means $^{w}\bm{\omega} =

\Omega \tilde{^{w}{\bm{x}}} = \Omega {^{w}{\bm{x}\_p}}$, also we know two vectors ${^{w}{\bm{x}\_p}}$ and

${^{w}{\bm{x}\_h}}$ are parallel. It has been proved that $^{h}\bm{x}\_h$ and $e\_{11}$ are eigenvector and eigenvalue

of matrix $^{h}\bm{E}$ respectively \citep{mahoney2011velocity}. The transformation matrix $^{w}\bm{R}\_h$

will not affect the eigenvalue ($e\_{11}$) but it will rotate the eigenvector ($^{h}\bm{x}\_h$) from the helix coordinate

frame to the global frame ($w$). As a result ($e\_{11}$) and ($^{w}\bm{x}\_p$) are the eigenvalue and eigenvector

in the world coordinate system respectively. By considering the vectors $^{w}\bm{x}\_p$ and $^{h}\bm{x}\_p$ are parallel

and definition of eigenvalue and eigenvector \footnote{Assume $A $ is a square matrix $n \times n$, we call

$\lambda$ an eigenalue of matrix $A$ if the non-zero vector $\bm{V}$ exists such that $A\bm{V} = \lambda \bm{V}$.

The vector $V$ is called eigenvector corresponding to eigenvalue $\lambda$.

\citep{Doe:2013Nov:Online}.}

the first componet of the desired velocity (${^{w}\bm{E}} {^{w}\bm{\omega}}$) can be

represented in the principle

coordinate system as follow:

\begin{equation}

{^{w}\bm{E}} {^{w}\bm{\omega}} = {^{w}\bm{E}} \Omega {^{w}{\bm{x}\_p}} = e\_{11} \Omega {^{w}{\bm{x}\_p}}

\label{E\_W}

\end{equation}

The similar reasoning has been used to represent the second componet (${^{w}\bm{D}} {^{w}\bm{f}}$) of

the velocity equation (\ref{first\_lineOf\_ propulsion method4}) in terms of the

principle coordinate system. In this case, $d\_{11}$ and $d\_{22}$ are eigenvalues of the matrix $^{h}\bm{D}$

such that $^{h}\bm{x}\_h$ is the eigenvector corresponding to the $d\_{11}$ and $d\_{22}$ is associated with

an eigenspace \footnote{Let $A $ be a $n \times n$ square matrix with an

eigenvalue $\lambda$. Then the union of all eigenvectors associated with the eigenvalue $\lambda $ and

vector zero is a subspace of $\Re ^{3}$ which is called the eigenspace for the eigenvalue $\lambda$.

\citep{Doe:2014Aug:Online}.} spanned by $\{ ^{h}\bm{y}\_{h} , ^{h}\bm{z}\_{h} \}$. Again, the eigenvalues and

eigenspace will remain unaffected under transformation matrix. Thus, the eigenvalue $d\_{11}$ is corresponding

to the $^{w}\bm{x}\_h$ and the eigenvalue $d\_{22}$ is related to the vector in the

subspace $\{ ^{w}\bm{y}\_h , ^{w}\bm{z}\_h \}$. In addition, the force vector can be decomposed into two

vectors; one parallel to the central axis of helix and the other perpendicular to that axis ($^{w}\bm{x}\_h$):

\begin{equation}

{^{w}\bm{f}} = \left ( (\bm{f} . \bm{x}\_h) ^{w}\bm{x}\_h \right) + \left ( (\bm{f} . \bm{y}\_h) ^{w}\bm{y}\_h

+ (\bm{f} . \bm{z}\_h) ^{w}\bm{z}\_h \right) = {^{w}\bm{f}}\_{\parallel h} + {^{w}\bm{f}}\_{\perp h}

\label{f\_Component\_globalAxis}

\end{equation}

If ${^{w}\bm{f}}\_{\perp h}$ deos not change then both ${^{w}\bm{f}}$ and ${^{w}\bm{f}}\_{\parallel h}$ will

not change if the microrobot rotate around its central axis. In addition, $^{w}\bm{y}\_p$ and $^{w}\bm{z}\_p$ are

in the eigenspace formed by $\{ ^{w}\bm{y}\_h , ^{w}\bm{z}\_h \}$. As a result, ${^{w}\bm{f}}\_{\perp h}$ can be written in the

principle coordinate frame as a linear combinations of two vectors ${^{w}\bm{z}}\_{p}$ and

${^{w}\bm{y}}\_{p}$. Beacuse $d\_{22}$ is the corresponding eigenvalue of any vector in the span of the

$\{ ^{w}\bm{y}\_h , ^{w}\bm{z}\_h \}$, so it will be the eigenvalue associated with ${^{w}\bm{f}}\_{\perp h}$.

Using the fact that $d\_{11}$ is the eigenvalue corresponding to $ {^{w}\bm{f}}\_{\parallel h} $ and implying

the transformation matrix we can write the force based on the principle component axis:

\begin{multline}

\qquad \qquad \qquad {^{w}\bm{D}} {^{w}\bm{f}} = {^{w}\bm{D}} {^{w}\bm{f}}\_{\parallel h} + {^{w}\bm{D}} {^{w}\bm{f}}\_{\perp h}

= d\_{11} {^{w}\bm{f}}\_{\parallel h} + d\_{22} {^{w}\bm{f}}\_{\perp h}\\

= d\_{11} \left ({\bm{f} . {\bm{x}}\_p } \right) {^{w}\bm{x}}\_p +

d\_{22} \left ({\bm{f} . {\bm{z}}\_p } \right) {^{w}\bm{z}}\_p \qquad \qquad

\label{force\_principle\_Components}

\end{multline}

Both components of the desired velocity are written on the basis of the principle components.

By replacing equations \ref{E\_W} and \ref{force\_principle\_Components} in equation \ref{first\_lineOf\_ propulsion method4} we have:

\begin{equation}

^{w}\bm{V} = d\_{11} \left ({\bm{f} . {\bm{x}}\_p } \right) {^{w}\bm{x}}\_p +

d\_{22} \left ({\bm{f} . {\bm{z}}\_p } \right) {^{w}\bm{z}}\_p + e\_{11} \Omega {^{w}{\bm{x}\_p}}

\label{velocity\_based\_principle}

\end{equation}

Therefore, non of the component of the velocity will change when the microrobot rotates around the central

axis.

Since ${\| \bm{{V}\_{hor}}\|} \neq 0$, as it is shown in the Fig \ref{horiz\_verti\_velocity} we can define

the angle $\alpha$ between the vector $\bm{v}$ and the vertical axis in the world frame.

\begin{equation}

\alpha = {\tan}^{-1} ({\| \bm{V}\_{hor} \|} / {\| \bm{V}\_{ver} \|})

\label{alpha\_velocity}

\end{equation}

The microrobot is required to be in a position above the desired velocity vector (upward) with the angle $\psi$

to compensate for the gravity vector. If we project the desired velocity equation (\ref{velocity\_based\_principle})

into principle coordinate axis then we have:

\begin{equation}

(\bm{V} . \bm{x}\_p) = d\_{11} \left ({\bm{f} . {\bm{x}}\_p } \right) + e\_{11} \Omega

\label{Xp\_velocity}

\end{equation}

\begin{equation}

(\bm{V} . \bm{z}\_p) = d\_{22} \left ({\bm{f} . {\bm{z}}\_p } \right)

\label{Xz\_velocity}

\end{equation}

As can be seen in the Fig \ref{horiz\_verti\_velocity}, both sides of the

equation \ref{Xz\_velocity} can be replaced by its equivalents:

\begin{equation}

(\bm{V} . \bm{z}\_p) = - {\| \bm{V} \|} \sin(\psi)

\label{Xz\_velocity\_equival}

\end{equation}

\begin{equation}

(\bm{f} . \bm{z}\_p) = {\| \bm{f} \|} \sin(\psi - \alpha)

\label{Xz\_velocity\_equivali}

\end{equation}

Thus, the replaced equation will lead to the following:

\begin{equation}

- {\| \bm{V} \|} \sin(\psi) = d\_{22} {\| \bm{f} \|} \sin(\psi - \alpha)

\label{finding\_psi}

\end{equation}

by applying the subtraction law for $ \sin(\psi - \alpha)$ \footnote{$\sin(\psi - \alpha) =

\sin(\psi) \cos(\alpha) - \cos(\psi) \sin(\alpha)$},

the angle $\psi$ can be optained from the following

equation:

\begin{equation}

{\psi} ={{\tan}^{-1}}

\frac{\left( d\_{22} {\| \bm{f} \|} \sin(\alpha) \right)}{ \| {\bm{V} \| + d\_{22} \| {\bm{f}} \|} \cos(\alpha) }

\label{psi}

\end{equation}

All the parameters in the above equation are known and the direction point ($\tilde{\bm{X}}$) of the microrobot

can be reconstructed by using angles $\alpha$ and $\psi$ and defining a dummy vector $\tilde{\bm{V}}$ such

that $\tilde{\bm{V}} = {\tilde{\bm{V}}}\_{ver} + {\tilde{\bm{V}}}\_{hor} $ where

${\tilde{\bm{V}}}\_{ver} = - \| {\tilde{\bm{V}}}\_{hor} \| \tan(\pi /2 - \alpha + \psi) \hat{\bm{g}}$ and

${\tilde{\bm{V}}}\_{hor} = {{\bm{V}}}\_{hor}$. Therefore the final solution for the direction point is:

\begin{equation}

\tilde{\bm{X} } = \frac{\tilde{\bm{V}}}{\| {\tilde{\bm{V}}} \|}

\label{direction\_point}

\end{equation}

Therefore the angular velocity ($\Omega$) will be derived from equation \ref{Xp\_velocity}, considering that

$(\bm{V} . \bm{x}\_p) = \| {\bm{V}} \| \cos({\psi})$ and $({\bm{f} . {\bm{x}}\_p }) = - \| {\bm{f}} \| \cos(\psi - \alpha) $ :

\begin{equation}

\Omega = \frac{\| {\tilde{\bm{V}} \| \cos(\psi) } + d\_{11} \| \bm{f} \| \cos(\psi - \alpha)}{e\_{11}}

\label{FinalAngular\_velo}

\end{equation}

At this point the rotational velocity of microrobot can be used to compute the magnetic torque according to

the following equasion from propulsion equasion system;

\begin{equation}

\tau = \bm{B} \bm{V} + \bm{C} \Omega

\label{finalTorque\_rotation}

\end{equation}

Where $\bm{V}$ is known and $\bm{B}$ and $\bm{C}$ are precomputed from coeffient matix. The torque in the

magnetic torque equasion is replaced by its equivalent \ref{finalTorque\_rotation};

\begin{equation}

\tau = \bm{V}M \times \bm{B}

\label{finding-B}

\end{equation}

Where $M$ magnetisation constant and $V$ is a volume of the magnetic object. Finally, the electric current

($i$) is required to generate a dynamic magnetic field is achieved by the following;

\begin{equation}

|\bm{B}| = (\frac{b^2}{(b^2+l^2)^{3/2}}){\mu}\_0 i

\label{Current}

\end{equation}

And the simulation algorithm is completed.

\begin{comment}

%%%%%%%%%

%%%%%%%%%%%%%%%%%%%% Testing %%%%%%%%%%%%%

%%%%%%%%%

\chapter{Experiments}

% Table show the range of wireless microrobopts input frequency and their speed.

Fe-ABF and FeTi-ABFs are

two magnetic fields of strenghts 1mT and 3 mT

The test show that forward speed is increasing with increasing input frequency. The maximum speed of Fe-ABF was

48.9 $\mu ms^{-1}$ at the field strength of $9 mT$ and $72 Hz$ ~\citep{qiu2014noncytotoxic}.

\end{comment}

%%%%%%%%%

%%%%%%%%%%%%%%%%%%%% Results %%%%%%%%%%%%%

%%%%%%%%%

\chapter{Results}\label{result}

\section{Simulation}

The aim of implementing the simulation framework for a microhelix is to analyse the effect of

the key parameters on a microhelix\rq{}s performance. As the microhelix moves in a fluid environment, modeling its swimming

motion is one aspect of the implementation.

\paragraph{Simulation software}

Initially, Matlab was selected as the simulator software. However, after exploring the different aspects

of the simulation, it became evident that the limitations of the software made it impossible to develop a complete simulation

framework. One of the limitations was inability of the software to incorporate and bind all the physics involved in this

modelling. Furthermore, Matlab was incapable of considering all aspects of a fluid environment. Therefore,

another simulation software called COMSOL was used to implement the model. Although, COMSOL offers

many build in environment that helps to make our simulation model, implementing an entire framework was

required many considerations.

\paragraph{Model components}

The system configuration (figure \ref{Simulation framework}) is based on the experimental setup

run by \citeauthor{mahoney2011velocity}. Although the experiment was run on a mili-robot, they used three inset Hemholtz

coils to generate a magnetic field for microrobot actuation.

The table \ref{Simultion model configration} represents the details

of the Helmholtz coils which generates a dynamic magnetic field by using AC current. The size of

fluid box is $25 (mm) \times 25(mm) \times 25 (mm)$ and the viscosity of fluid inside the box is $2000 (\frac{N}{m^2} s)$

which is the viscosity of the corn syrup.

%--------------- Table of configration model of the microrobots------------------------

\begin{table}[!ht]

\centering% used for centering table

{\rowcolors{2}{gray!50}{gray!50!blue!13}

\begin{tabular}{c c c }% centered columns (8 columns)

\toprule[2.0pt]

\head{Coil set} & \head{Coil radius (mm)} & \head{Number of wraps} \\

%heading

\midrule

%\hline% inserts single horizontal line

Inner & 44 & 63 \\% inserting body of the table

Middle & 69 & 99 \\

Outer & 98 & 143 \\[1ex]% [1ex] adds vertical space

\bottomrule[2.0pt]

\end{tabular}

}

\label{Simultion model configration}% is used to refer this table in the text

\caption[Simultion model configration]{Simultion model configuration. The detail of three inset Helmholt coils setup

\citep{mahoney2011velocity}.}\label{Simultion model configration}% title of Table

\end{table}

%-------------------------------------------------------------------------------------------

Simulation forms of two main parts; microhelix propulsion mechanism and its

actuation method. The model is made of three components;

\begin{itemize}

\item Three inset Helmholtz coils

\item The fluid box

\item The microhelix

\end{itemize}

In order to simplify the model, it was broken down into three sub-models. Each sub-model made of two

components and the entire model made by combining three sub-models as shown

in figure \ref{Simulation framework}. In each sub-model, the physics involve in its componets were solved.

For example, in sub-model 1 we studied the effect of the magnetic field on the fluid box without considering the microhelix

inside the box. Defining an appropriate domain for the solution is an important factor in modelling

because the magnetic filed decays as the distance from the magnetic source increases. Thus, the sphere domain with the radius of $3 cm$

defined to solve the model as it is shown in figure \ref{Simulation domain}

%--------------- Table of configration model of the microrobots------------------------

\begin{table}[!ht]

\centering% used for centering table

{\rowcolors{2}{gray!50}{gray!50!blue!13}

\begin{tabular}{c c c c}% centred columns (4 columns)

\toprule[2.0pt]

\head{Sub-models} & \head{Helmholtz coils} & \head{Fluid box} & \head{Microhelix} \\

%heading

\midrule

%\hline% inserts single horizontal line

1 & \checkmark & \checkmark & \\

2 & & \checkmark & \checkmark \\

3 & \checkmark & & \checkmark \\[1ex]% [1ex] adds vertical space

\bottomrule[2.0pt]

\end{tabular}

}

\label{Modeling Simulation Componets }% is used to refer this table in the text

\caption[Modeling Simulation Componets]{Modeling Simulation Componets. The table shows the componets of each sub-model.}

\end{table}

%-------------------------------------------------------------------------------------------

\begin{figure}

\centering

\begin{subfigure}[b]{0.43\textwidth}

\includegraphics[width=\textwidth]{simulation}

\caption{Simulation framework}

\label{simulation}

\end{subfigure}~~

\begin{subfigure}[b]{0.565\textwidth}

\includegraphics[width=\textwidth]{helix-in-box}

\caption{Helix in the box}

\label{helix-in-box}

\end{subfigure}

\caption[Simulation framework]{Simulation framework. (a) The framework consists of the three inset coils,

a fluid bux with a microhelix inside (b).}\label{Simulation framework}

%add desired spacing between images, e. g. ~, \quad, \qquad, \hfill etc.

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%add desired spacing between images, e. g. ~, \quad, \qquad, \hfill etc.

%(or a blank line to force the subfigure onto a new line)

\end{figure}

The effect of microhelix design parameters such as helical angle on rotational velocity of microhelix was examined

in a simulation. The considered parameters are;

\begin{itemize}

\item Helix angle

\item Helix pitch

\item Helix radius

\item Helix filament radius

\end{itemize}

\begin{figure}

\centering

\begin{subfigure}[b]{0.48\textwidth}

\includegraphics[width=\textwidth]{simulationDomain}

\caption{Simulation domain}

\label{simulation}

\end{subfigure}~~

\begin{subfigure}[b]{0.48\textwidth}

\includegraphics[width=\textwidth]{arrow\_magnetic}

\caption{Magnetic flux density}

\label{helix-in-box}

\end{subfigure}

\caption[Simulation domain]{Simulation domain. (a) The sphere domain is defined for solving the model so the volume of

effective magnetic flux in the model can be obtained (b).}\label{Simulation domain}

%add desired spacing between images, e. g. ~, \quad, \qquad, \hfill etc.

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%add desired spacing between images, e. g. ~, \quad, \qquad, \hfill etc.

%(or a blank line to force the subfigure onto a new line)

\end{figure}

The result of each parametere\rq{}s effect on the rotational velocity of the microhelix is

represented as follows. The curve in the figure \ref{RV\_pitchAngle} describes the relationship between

the helical angle and rotational velocity.

The helix with a small helical angle will generate a small rotational velocity as shown in the figure \ref{RPitch}.

However, the simulator does not respond to a helical angle greater than $1 (rad)$. Interestingly, for the

zero helical angle, the simulator shows the rotational velocity just under $12 (rad/s)$. This confirms that

in a extremely small helical angle \ac\*{RFT} treats a helix as a cylinder. and therefore calculates a

rotational velocity of a cylinder.

Figure \ref{RV\_helixPitch} describes the relationship between a rotational velocity of microhelix and its

pitch. This curve shows an increase in rotational velocity by increasing a helix pitch. Figures \ref{RV\_pitchAngle} and

\ref{RV\_helixPitch} demonstrate the impact of helix radius and filament radius on rotational velocity respectively.

Therefore, helical radius is more important than the filament radius.

\begin{figure}

\centering

%\begin{wrapfigure}{r}{0.5\textwidth}

% \begin{center}

\includegraphics[width=1.00\textwidth]{RPitch}

\caption[Rotational velocity vs. helix angle]{Rotational velocity vs. helix angle}

\label{RV\_pitchAngle}

%\end{center}

%\end{wrapfigure}

\end{figure}

\begin{figure}

\centering

%\begin{wrapfigure}{r}{0.5\textwidth}

% \begin{center}

\includegraphics[width=0.90\textwidth]{RHpitch}

\caption[Rotational velocity vs. helix pitch]{Rotational velocity vs. helix pitch }

\label{RV\_helixPitch}

%\end{center}

%\end{wrapfigure}

\end{figure}

The figure \ref{RV\_helixRadius1} describes the reverse relation between the helix radius and rotational

velocity. As the radius of the helix is increasing the rotational velocity is decreasing, the highest rotational

velocity is just above $14 rad/s$ for the microhelix with $2 \mu m$ radius. A similar behaviour is observed

in figure \ref{RV\_filamentRadius} which is decreasing in rotational velocity by increasing the filament radius.

However, the range of figures in the rotational velocity axis shows the helix filament radius only has

a minor impact on the rotational velocity. Whilst, the considerable changes in rotational velocity occurs

by changing the helix angle and helix radius (figurs \ref{RV\_helixRadius1}, \ref{RV\_pitchAngle}).

\begin{figure}

\centering

%\begin{wrapfigure}{r}{0.5\textwidth}

% \begin{center}

\includegraphics[width=0.90\textwidth]{RRadius}

\caption[Rotational velocity vs. helix rediu]{Rotational velocity vs. helix redius }

\label{RV\_helixRadius1}

%\end{center}

%\end{wrapfigure}

\end{figure}

\begin{figure}

\centering

%\begin{wrapfigure}{r}{0.5\textwidth}

% \begin{center}

\includegraphics[width=0.90\textwidth]{RFRadius}

\caption[Rotational velocity vs. helix filament radius]{Rotational velocity vs. helix filament radius }

\label{RV\_filamentRadius}

%\end{center}

%\end{wrapfigure}

\end{figure}

There is a linear relationship between rotational and translational velocity of microhelix as it is shown

in figure \ref{Rotational velocity vs. translational velocity}.

\begin{figure}

\centering

%\begin{wrapfigure}{r}{0.5\textwidth}

% \begin{center}

\includegraphics[width=0.80\textwidth]{Trans\_Rot\_}

\caption[Rotational velocity vs. translational velocity]{Rotational velocity vs. translational velocity.

There is a linear realation between rotational and translational velocity.}

\label{Rotational velocity vs. translational velocity}

%\end{center}

%\end{wrapfigure}

\end{figure}

%simulation result

\begin{figure}

\centering

%\begin{wrapfigure}{r}{0.5\textwidth}

% \begin{center}

\includegraphics[width=0.90\textwidth]{magnetic\_field.png}

\caption[Microhelix first position]{Microhelix first position. The simulator finds and positions

the microhelix at the point of the strongest magnetic flux density.}

\label{Microhelix first position}

%\end{center}

%\end{wrapfigure}

\end{figure}

\section{Fabrication}

The design of the microrobot in this study is focused on the microswimmer with helical shape tail

and a possible propeller as the head that is attached to the helix body. Therefore, after studying the

key characteristic of the helix and identifying effective parameters a series of designs were produced. We started

by reproducing the previous design that had been made by other researchers in this field and finally proposed a

new design for the helical shape microswimmers. The software called Solidwork has been used for

designing and nanoscribe technology for the fabrication stage. In the following sections we present

each design and the fabricated result.

\paragraph{Circle base filament}

A common design for the helix was one with a filament having a circular base. The designs vary in terms of

changing the filament radius, helix pitch, helix length and helix radius. Some of the designs were successfully fabricated

as shown in figure \ref{Circle base helix}. The figure \ref{Damaged structures} presented a faulty result which is as result of either applying an insufficient or

excessive power laser beam in the fabrication process. Also another type of faulty result can be seen in figure \ref{smallPitchCircle}

which is due to excessive laser power.

\begin{figure}

\centering

\begin{subfigure}[b]{0.50\textwidth}

\includegraphics[width=\textwidth]{simpleHelixSolid}

\caption{Design of circle base helix}

\label{simpleHelixSolid}

\end{subfigure}~

\begin{subfigure}[b]{0.48\textwidth}

\includegraphics[width=\textwidth]{CircleBase}

\caption{Fabricated circle base helix}

\label{CircleBase}

\end{subfigure}

\caption[Circle base helix]{Circle base helix. This is the helix with $12 \mu m$ length and $4\mu m$ pitch. (a) Helix in design

stage and (b) shows the fabricated result.}\label{Circle base helix}

%add desired spacing between images, e. g. ~, \quad, \qquad, \hfill etc.

%(or a blank line to force the subfigure onto a new line)

%add desired spacing between images, e. g. ~, \quad, \qquad, \hfill etc.

%(or a blank line to force the subfigure onto a new line)

\end{figure}

\begin{figure}

\centering

\begin{subfigure}[b]{0.40\textwidth}

\includegraphics[width=\textwidth]{highLaser}

\caption{Damaged structre}

\label{Damaged structre}

\end{subfigure}~

\begin{subfigure}[b]{0.595\textwidth}

\includegraphics[width=\textwidth]{smallPitchCircle}

\caption{Small pitch helix}

\label{smallPitchCircle}

\end{subfigure}

\begin{subfigure}[b]{0.45\textwidth}

\includegraphics[width=\textwidth]{sphereFabricated}

\caption{Helix with a sphere head}

\label{sphereFabricated}

\end{subfigure}~

\begin{subfigure}[b]{0.53\textwidth}

\includegraphics[width=\textwidth]{largePitch}

\caption{Large pitch helix}

\label{largePitch}

\end{subfigure}

\caption[Fabricated structures]{The structure (a) is damaged as a result of high

laser beam and structure (b) has a small pitch $1.3 \mu m$ and the result is not satsfactory. The structure

with the sphere head (c) is fabricated and the helix with the larger pitch (d) is clearly shown,}\label{Damaged structures}

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%(or a blank line to force the subfigure onto a new line)

%add desired spacing between images, e. g. ~, \quad, \qquad, \hfill etc.

%(or a blank line to force the subfigure onto a new line)

\end{figure}

\paragraph{Rectangle base filament}

\begin{figure}

\centering

\begin{subfigure}[b]{0.566\textwidth}

\includegraphics[width=\textwidth]{constant-pitch}

\caption{Constant pitch helix}

\label{constant-pitch}

\end{subfigure}~

\begin{subfigure}[b]{0.42\textwidth}

\includegraphics[width=\textwidth]{variable-pitch1}

\caption{Variable pitch helix. }

\label{variable-pitch1}

\end{subfigure}

\caption[Variable pitch helix]{Variable pitch helix. The image (a) is shown the constant pitch helix which could not printed

vertically. However, the image (b) represents the successful vertically fabricated of (a) by making its pitch

variable.}\label{Pitch variable}

%add desired spacing between images, e. g. ~, \quad, \qquad, \hfill etc.

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%add desired spacing between images, e. g. ~, \quad, \qquad, \hfill etc.

%(or a blank line to force the subfigure onto a new line)

\end{figure}

The second design has the rectangle base filament. This design can make two different helix shapes depending on which side of

the rectangle is revolved around the spiral path in the design stage. The

figure \ref{Rectangle based structures} shows a variety of designs

based on the rectangle filament with different pitch, length and helical radius.

\begin{figure}

\centering

\begin{subfigure}[b]{0.49\textwidth}

\includegraphics[width=\textwidth]{Cursor}

\caption{Small pitch rectangle base filament}

\label{Rectangle base filament}

\end{subfigure}~

\begin{subfigure}[b]{0.505\textwidth}

\includegraphics[width=\textwidth]{vertical-rentanglebased}

\caption{Vertically fabricated ractangle based helix}

\label{vertical-rentanglebased}

\end{subfigure}

\begin{subfigure}[b]{0.59\textwidth}

\includegraphics[width=\textwidth]{ribbon}

\caption{Rectangle based helix with large length}

\label{ribbon}

\end{subfigure}~

\begin{subfigure}[b]{0.395\textwidth}

\includegraphics[width=\textwidth]{threeHelix}

\caption{Revolved }

\label{threeHelix}

\end{subfigure}

\begin{subfigure}[b]{0.50\textwidth}

\includegraphics[width=\textwidth]{closeRectangle}

\caption{The rectangle base}

\label{closeRectangle}

\end{subfigure}~

\begin{subfigure}[b]{0.475\textwidth}

\includegraphics[width=\textwidth]{collaps}

\caption{Collapsed structures}

\label{collaps}

\end{subfigure}

\caption[Rectangle based structures]{Rectangle based structures. The structures (a) and (b) are examples of

revolving the smaller side of the rectangle around the central axis of helix. The structure

can be printed vertically (b). The structures with a small pitch (a) did not print ideally whilst

the one with larger pitch (d) did print clearly. The printed structure can collapse on each other (f).

Highly zoomed image (e) shows the rectangle base of the

helix.}\label{Rectangle based structures}

%add desired spacing between images, e. g. ~, \quad, \qquad, \hfill etc.

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%add desired spacing between images, e. g. ~, \quad, \qquad, \hfill etc.

%(or a blank line to force the subfigure onto a new line)

\end{figure}

\paragraph{Pitch variable helix}

In order to be able coat the structures with magnetic material, the structures should be printed vertically.

The new helix design produced has a variable pitch. The problem with previous designs was that the starting helix

angle from the bottom of the helix was too high. Therefore, the structure had a poor surface to rely on when in a vertical

position. The first solution was producing structures with a small helix pitch to ensure a stronger base for vertical

printing. However, as is shown in the fogure \ref{collaps} part (a) the overall result of the helix with the small pitch

is not satisfactory. Thus, the idea of designing a helix with variable pitch enables us to meet both the simultaion and fabrication requirements. The

other advantage of having the variable pitch is providing a stronger base for a microhelix with an attached

propeller. The result of the new design is shown in figure \ref{Pitch variable}.

%%%%%%%%%

%%%%%%%%%%%%%%%%%%%% Discussion %%%%%%%%%%%%%

%%%%%%%%%

\chapter{Discussion}\label{discussion}

\paragraph{}

The design of microhelix is based on the key characteristics of the helical shape. The circular shape was often used for the filament base

of a helix. Therefore, the initial design was a helix with a circle base filament which was then

integrated with different propeller heads such as a sphere and a square. Changing each characteristic one at a time and

keeping the rest of the characteristics constant optimised the design of the microhelix. This algorithmic process was repeated

for all new designs, such as a helix with a rectangle base filament. The helix design was optimised

in terms of both fabrication and simulation.

In the fabrication process, the nanoscribe technology was used for 3D printing in micro size. This

technology is based on lithography system to write a microstructure. The result of fabricated structures

showed that the required laser power varies for printing each microhelix design. The result of horizontally printing

microhelix was successful in many cases. However, in a few cases the microstructure collapsed because the laser power

applied was either too high or too low. The appropriate amount of laser power was design dependent and so had to be varied for

each design. In some cases the microstructure also collapsed during the developing process. The vertically fabricating

microhelix was a challenge, as most of the design did not provide sufficient contact surface to support their

weight. The new helix design with a variable pitch provided the structure with sufficient contact area with the

substrate. The variable pitch helix has the smaller pitch at the bottom and a larger pitch over the rest of the

body. As a result a small pitch at the bottom satisfies the vertically fabricated requirement of microhelix

and larger pitch on the rest of the helix body helps the swimming motion of the microhelix.

\paragraph{}

To model the swimming motion of the microhelix, three models were studied and \ac\*{RFT} were

implemented in the simulator. \ac\*{RFT} has ignored the

hydrodynamic interaction between the fluid flow produced by different segments of the microhelix.

Therefore, in the microhelix with the smaller pitch the interaction between various parts of the helix

increased and the helix is converted into a cylinder. This issue might result in an error when predicting

the force and torque in a microhelix with an extremely small pitch. Thus, the variable pitch microhelix will help to avoid this issue.

To model the swimming motion of the microhelix three models were studied and \ac\*{RFT} were

implemented in the simulator.

Each microhelix propulsion model segmented the filament of the

microhelix differently. Then the model analyses an interaction of a small segment of

the filament with the hydrodynamic characteristics of a high viscous fluid to achieve non fluidic force, torque and drag

acting on a helix body. All three propulsion models were only applied to the microhelix with a circle base

filament. Therefore, we will not be able to implement the propulsion model for other designs such

as microhelix with a rectangle base filament.

\paragraph{}

Magnetic field is a safe power source to be used for actuating microrobot in the fluidic environment.

There are advantages to applying torque driven magnetic field over force driven magnetic field, which makes it a preferable

approach. Torque driven method can be applied on either microrobot with the flexible tail or a rigid

tail. This method is more efficient than force driven as the rotation of helix leads to translational

movement in the fluidic environment whilst the force driven method pulls the helix to generate

the translational velocity.

\paragraph{}

Implementing the model using simulation software is a challenge. There is very few simulation

software available that provide all the requirements of the model. Therefore, modelling the simulator for microrobot

navigation becomes more complex. For example the hydrodynamic property of the fluid and its

effect on the swimming microrohelix cannot be modelled entirely with Matlab. COMSOL software

offers more advanced simulation in terms of solving the multiphysics model by coupling the different

physics. However, the modelling can be performed differently by coupling different components

of the system.

%%%%%%%%%

%%%%%%%%%%%%%%%%%%%% Conclusion %%%%%%%%%%%%%

%%%%%%%%%

\chapter{Conclusion and future work}

The magnetically actuated helical shape microrobot has the advantage that it can be used in both

in vivo or in vitro applications. Different microhelix shapes were designed, which included designs

reproduced from previous researchers as well as new designs being presented. Of the latter group,

the most successful new design was a helix with a variable pitch. This was because this design provided greater

contact area with the substrate and this in turn enabled the structure to stand vertically during the fabrication process.

The stronger base also gave the variable pitch design an advantage when attaching a propeller to the microrbot.

The new design performed satisfactorily during the simulation process.

The \ac\*{RFT} was used as a locomotion model for simulating the motion of a remotely controlled

microrbot. The other propulsion models presented are \ac\*{SBT} and \ac\*{RSM}, which also describe the

motion of a microhelix but were also remotely controlled for the first time.

According to the simulation results, the helix angle is the most important characteristic of the helix, in terms of its impact on rotational and

translational velocity. The other characteristic of helix radius has a lesser impact.

The fabrication process performed by nanoscribe facility and microstructures were observed

under \ac\*{SEM}.

Initially Matlab was used for implementing the model and towards the end of the project the

COMSOL Multiphysics software was used because of the limitations of Matlab in solving the model.

\paragraph{Future work}

In order to validate the results of this study we need to run an experiment in a laboratory

to compare the result of the simulation with the experiment. The simulation framework is

needs to be further optimised in order to simulate microhelix in fluids with different viscosity. In terms

of fabrication, the effects of using a combination of magnetic material for microhelix coating can

be assessed. The ideal locomotion method for a microrobot requires further investigation.

The design of an advanced microrobot is application dependent.

The possibility of actuating a microrobot with other power sources such as ultrasonic can be

considered. Furthermore, the fundamental design of a microrobot can be integrated with other tools to build

an advanced microrobot. The microhelix with its transport claw is already proposed and demonstrated their

result in moving micro objects. A swarm of microrobots and a mechanism for controlling them individually will

improve their performance.

Therefore, researching current medical applications and limitations of such surgical

tools can lead to more advanced microrobot designs for specific applications.

%%%%%%%%%%%%%%%%% List of the references are not used directly in the text %%%%%%%

\nocite{lauga2006swimming}

%%%%%%%%%%%%%%

%\renewcommand{\bibname}{References}

\bibliographystyle{unsrtnat}

%\bibliographystyle{plainnat}

\bibliography{References}

%\addcontentsline{toc}{chapter}{\numberline{}References}

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